Quiz 1

Name: Key
Date: 1 - 24 - 14

Directions: Show all work for full credit.

Problem 1) The table shows the position of a cyclist.

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) (meters)</td>
<td>0</td>
<td>1.4</td>
<td>5.1</td>
<td>10.7</td>
<td>17.7</td>
<td>25.8</td>
</tr>
</tbody>
</table>

(a) Find the average velocity for each time period:

(i) \([1, 3]\) (ii) \([2, 3]\) (iii) \([3, 5]\) (iv) \([3, 4]\)

(b) Approximate the instantaneous velocity when \( t = 3 \).

Solution:

(a) Here we need to find the average rates of change over each time interval. To do this we use version 1 of the difference quotient, that is \( \frac{y_1 - y_2}{x_1 - x_2} \). Here our \( x \) values are seconds and our \( y \) values are meters. Hence (i) \( \frac{10.7 - 1.4}{3 - 1} = 4.65 \), (ii) \( \frac{10.7 - 5.1}{3 - 2} = 5.6 \), (iii) \( \frac{25.8 - 10.7}{5 - 3} = 7.55 \), (iv) \( \frac{17.7 - 10.7}{4 - 1} = 7 \).

(b) From (a) the approximate instantaneous velocity, or instantaneous rate of change is between the average velocities found from (ii) and (iv). Hence the instantaneous velocity \( v \) is \( 5.6 \leq v \leq 7 \), I will also accept \( v \approx 6.3 \).

Q.E.D
Problem 2) Sketch the graph of the function

\[ f(x) = \begin{cases} 
1 + x & \text{if } x < -1 \\
x^2 & \text{if } -1 \leq x < 1 \\
2 - x & \text{if } x \geq 1 
\end{cases} \]

and determine the values \( a \) for which \( \lim_{x \to a} f(x) \) exists. [Hint: Find where the limit does not exists first.]

Solution: I trust you can graph this.

Notice that the only place where the graph is not continuous is at \( x = -1 \), where \( \lim_{x \to -1^-} = 0 \) while \( \lim_{x \to -1^+} = 1 \), the left and right handed limits do not match only at this point. So the limit exists everywhere except at \( x = -1 \).

Q.E.D