

Some comments on pre-calculus*

C. L. Zirbel
Department of Mathematics and Statistics
Bowling Green State University
Bowling Green, Ohio 43402

April 13, 2009

Contents

1	<i>Introduction</i>	1
2	<i>Exponentials and logarithms</i>	2
2.1	<i>Logarithms</i>	2
2.2	<i>Inverses of logarithms (alternate section title)</i>	2
8	<i>Polynomials</i>	2
A	<i>Appendix</i>	2

1 Introduction

Pre-calculus consists of algebra and trigonometry. My favorite equation from algebra is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{1}$$

and my favorite equation from trigonometry is

$$x^2 + y^2 = 1,$$

which doesn't look like much until you remember that

$$\begin{aligned} x &= \cos(\theta) \\ y &= \sin(\theta). \end{aligned} \tag{2}$$

Note that the previous sentence extended over several displayed equations, with proper punctuation in each. (You should always type something after an equation, otherwise there is too much white space before whatever comes next.)

*Electronic version available at personal.bgsu.edu/~zirbel/592

2 Exponentials and logarithms

These are some of the most important functions in all of mathematics. See [1, Chapter 4]. Unfortunately, many people do not like them very much.

2.1 Logarithms

I learned in Calculus II that the natural logarithm can be defined in terms of an integral:

$$\ln(x) = \int_1^x \frac{1}{t} dt, \quad x > 0. \quad (3)$$

(See [2] for more information.) Note that it is important to state the domain of definition. Also, I always like to put parentheses around the arguments of functions like natural logarithm, sine, and cosine, cf. Equation 2.

Theorem 1. *The function \ln is strictly increasing and continuous. Its range is $(-\infty, \infty)$.*

Proof. We omit the proof for brevity. □

Remark 2. *One often makes remarks after theorems. In this article, remarks are numbered separately from equations.*

2.2 Exponentials

Once you show that the function \ln is strictly increasing, continuous, and has range $(-\infty, \infty)$, you can define the exponential function as its inverse.

8 Polynomials

Section 2 discussed exponentials and logarithms. Now we turn to something easier: polynomials. The zeros of quadratic polynomials can be found using Equation 1 on page 1.

A Appendix

This \LaTeX document contains several types of cross-references, references to equations, to pages, and to cited works. Each equation or cited work that will be cited needs to be assigned an alphanumeric tag, but \LaTeX automatically numbers equations, pages, and citations. That way, if you add more later, you need not fix all the cross-references. Note that \LaTeX needs to be run more than once to get all the references straight. The program `texify` automatically runs \LaTeX as many times as it takes.

To produce PDF output with hyperlinks, either use `pdflatex` or, if you are using `.eps` graphics, using \LaTeX , then `dvips`, then `ps2pdf`. The only problem is that there is no button in LEd for `ps2pdf`, so you need to right click the `.ps` file, Open With, then Browse to find the batch file `ps2pdf.bat`, which is in the `MiKTeX/miktex/bin` folder.

References

- [1] *Precalculus: Mathematics for Calculus*, 3rd edition, by James Stewart, Lothar Redlin, Saleem Watson. Brooks/Cole Publishing Company, Pacific Grove, California. 1998.
- [2] *Single-Variable Calculus with Analytic Geometry*, 4th edition, by C. H. Edwards, Jr. and David E. Penney. Prentice Hall, Englewood Cliffs, New Jersey. 1994.