MATHEMATICS 445/545 - APPLIED PROBABILITY - FALL 2000 - ZIRBEL

Homework #4. Due Friday, October 6.

1. Suppose $X_1, X_2, \ldots$ all have the same mean and take values in $[0, \infty)$. Let N take values in the set $\{0, 1, 2, \ldots\}$ and suppose that N is independent of X_i for $i = 1, 2, 3, \ldots$. Define a new random variable S by $S = X_1 + \cdots + X_N$; when it happens that $N = 0$, then we set $S = 0$.

The steps below show that, in this case, $\mathbb{E}[S] = \mathbb{E}[N]\mathbb{E}[X_1]$. Your job is to rewrite these steps and *provide justifications for each step.*

The notation $1_{\{N=n\}}$ means a random variable whose value is 1 when $N = n$ and 0 otherwise.

$$\begin{aligned}
\mathbb{E}[S] &= \mathbb{E}\left[\left(\sum_{n=0}^{\infty} 1_{\{N=n\}}\right) S\right] \\
&= \sum_{n=0}^{\infty} \mathbb{E}\left[1_{\{N=n\}} S\right] \quad \text{because } 1_{\{N=n\}} S \geq 0 \\
&= \sum_{n=0}^{\infty} \mathbb{E}\left[1_{\{N=n\}} (X_1 + \cdots + X_N)\right] \\
&= \sum_{n=0}^{\infty} \mathbb{E}\left[1_{\{N=n\}} (X_1 + \cdots + X_n)\right] \\
&= \sum_{n=0}^{\infty} \sum_{i=1}^{n} \mathbb{E}\left[1_{\{N=n\}} X_i\right] \\
&= \sum_{n=0}^{\infty} \sum_{i=1}^{n} \mathbb{E}\left[1_{\{N=n\}}\right] \mathbb{E}\left[X_i\right] \\
&= \sum_{n=0}^{\infty} \sum_{i=1}^{n} \mathbb{P}(N = n) \mathbb{E}\left[X_1\right] \\
&= \sum_{n=0}^{\infty} n \mathbb{P}(N = n) \mathbb{E}\left[X_1\right] \\
&= \mathbb{E}[N]\mathbb{E}[X_1]
\end{aligned}$$

This result was used in computing the expected waiting time in our discrete time queueing model.

2. For the discrete time queueing model we have been discussing in class, we found that $W = 1/(p - \lambda)$. Write a Matlab program called `wait.m` which produces a graph of this relationship. Add comments to each line of the program to explain what is being done. Print out the graph and the program.

Some suggested commands to use and how to get help on them:

`lambda = 0:0.01:p;`	help colon
`W = 1./(p-lambda);`	help slash
`plot(lambda,W);`	help plot
`hold on`	help hold

You should change the horizontal axis to display values out to $\lambda = 1$ using `axis`. Add a vertical dashed line at p using a command similar to `plot([p p], [0 10], '-');`. Add labels to the axes and a title for the graph.