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## Plenary Talks

### Subgroups of right-angled Artin groups

Ruth Charney

*Brandeis University*

charney@brandeis.edu

Coauthors: Michael Carr

It is well known that right-angled Artin groups (RAAGs) contain a wide variety of interesting, and sometimes exotic, subgroups. Yet a theorem of Baudisch from 1981 states that all 2-generator subgroups are either free or free abelian. I will talk about some recent results of my student Michael Carr showing that these 2-generator subgroups are always quasi-isometrically embedded in the RAAG and discuss consequences of this theorem for cubulating 3-manifold groups.

### CH and the Moore-Mrowka Problem

Todd Eisworth

*Ohio University*

eisworth@ohio.edu

Coauthors: Alan Dow

We will discuss the history of the Moore-Mrowka problem in general topology (Must compact spaces of countable tightness be sequential?), and then outline a proof that a positive answer is consistent with the Continuum Hypothesis. This is joint work with Alan Dow.

### Dehn fillings and elementary splittings of groups

Daniel Groves

*University of Illinois at Chicago*

groves@math.uic.edu

Coauthors: Jason Manning (Cornell)

Dehn fillings of 3-manifolds are a classical tool, and have been generalized to a coarse geometric setting in numerous ways. We are interested in Dehn fillings of relatively hyperbolic groups, which have seen many recent applications. Many basic properties of Dehn fillings remain mysterious, in particular what happens to topological properties of the (Gromov or Bowditch) boundary.

The existence of certain kinds of splittings of relatively hyperbolic groups can be detected via topological properties of the boundary. We prove that the non-existence of splittings persist under long Dehn fillings, and deduce properties

about the boundary under such fillings. One consequence recovers the (group-theoretic content of the) classical fact that long Dehn fillings of finite-volume hyperbolic 3-manifolds are irreducible.

### **Some applications of the Mountain Climbing Theorem to Continuum Theory**

Alejandro Illanes

*Universidad Nacional Autonoma de Mexico*

`illanes@matem.unam.mx`

The classic Mountain Climbing Theorem says that if  $f$  and  $g$  are piecewise linear maps from  $[0, 1]$  to itself such that  $f(0) = 0 = g(0)$  and  $f(1) = 1 = g(1)$ , then there exists piecewise linear maps  $h$  and  $k$  from  $[0, 1]$  to itself such that  $f \circ h = g \circ k$ . A continuum is a nonempty compact connected metric space. In this talk we will see that this theorem has had some applications for solving problems in Continuum Theory.

### **A nonamenable finitely presented group of piecewise projective homeomorphisms**

Yash Lodha

*EPFL, Lausanne*

`y1763@cornell.edu`

Coauthors: Justin Moore

In this talk, I will describe a finitely presented subgroup of Monod's group of piecewise projective homeomorphisms of the real line. This provides a new example of a finitely presented group which is nonamenable and yet does not contain a nonabelian free subgroup. The example is moreover torsion free and of type  $F_\infty$ . A portion of this is joint work with Justin Moore.

## Loops of transitive interval maps

Michał Misiurewicz

*Indiana University-Purdue University Indianapolis*

`mmisiure@math.iupui.edu`

Coauthors: Sergiy Kolyada (Institute of Mathematics, NASU) Lubomir Snoha (Matej Bel University)

In an earlier paper we investigated topology of spaces of continuous topologically transitive interval maps. Let  $\mathcal{T}_n$  denote the space of all transitive maps of modality  $n$ . For every  $n \geq 1$  we constructed in  $\mathcal{T}_n \cup \mathcal{T}_{n+1}$  a loop (call it  $L_n$ ), which is not contractible in this space. One of the main open problems left was whether  $L_n$  can be contracted in the union of more of spaces  $\mathcal{T}_i$ .

Since all elements of the loops  $L_n$  are maps of constant slope, we are studying the spaces  $\mathcal{TCS}_n$  of transitive maps of modality  $n$  of constant slope. We show that for every  $n \geq 2$  the loops  $L_n$  and  $L_{n+1}$  can be contracted in  $\mathcal{TCS}_n \cup \mathcal{TCS}_{n+1} \cup \mathcal{TCS}_{n+2}$ . Moreover, the loop  $L_1$  can be contracted in  $\mathcal{TCS}_1 \cup \mathcal{TCS}_2 \cup \mathcal{TCS}_4$ .

Additionally, we describe completely the topology (and geometry, in a certain parametrization) of the spaces  $\mathcal{TCS}_1$ ,  $\mathcal{TCS}_2$  and  $\mathcal{TCS}_1 \cup \mathcal{TCS}_2$ . In particular, we show that the space  $\mathcal{TCS}_1 \cup \mathcal{TCS}_2$  is homotopically equivalent to the circle.

## Mapping class groups of non-orientable surfaces

Luis Paris

*Universite de Bourgogne*

`lparis@u-bourgogne.fr`

Let  $M$  be a connected non-orientable surface. We denote by  $\text{Homeo}(M)$  the group of homeomorphisms of  $M$ . The *mapping class group* of  $M$ , denoted by  $\mathcal{M}(M)$ , is the group of isotopy classes of elements of  $\text{Homeo}(M)$ . This talk will be an introductory lecture for non-experts on mapping class groups of non-orientable surfaces. We will mainly present low genus examples, some particular elements lying in these groups, and few properties (for instance, about the automorphism groups).



## Semi-Plenary Talks

### **Finiteness properties of groups acting on buildings**

Peter Abramenko

*University of Virginia*

pa8e@cms.mail.virginia.edu

Among the most natural questions one can ask about a discrete countable group  $G$  is whether it is finitely generated or finitely presented. More generally, one would like to know whether  $G$  is of type  $F_n$  for some given natural number  $n$ . A standard method to get answers for these questions is to analyze the action of  $G$  on an appropriate space  $X$ . In this talk, I will concentrate on the case where  $X$  is a building or a product of buildings. Results for some classes of groups and open questions in this context will be presented.

### **Subsets of the cubic connectedness locus. II**

Alexander Blokh

*UAB*

ablokh@math.uab.edu

Coauthors: L. Oversteegen, R. Ptacek, V. Timorin

This talk continues the presentation by Lex Oversteegen. It is based on joint work with L. Oversteegen, R. Ptacek and V. Timorin.

### **Canonical cofinal maps on ultrafilters and properties inherited under Tukey reducibility**

Natasha Dobrinen

*University of Denver*

natasha.dobrinen@du.edu

Tukey reducibility between ultrafilters is determined by the existence of a map from one ultrafilter to another which takes each filter base of one ultrafilter to a filter base of the other. Such maps are a priori essentially maps on the powerset of the natural numbers, and thus there are  $2^c$  many possible cofinal maps. In some cases, though, we may find canonical forms of cofinal maps, which reduces the Tukey type of an ultrafilter to size continuum and allows for fine analysis of the structures of Tukey types. Building on a result of Dobrinen and Todorcevic which showed that each  $p$ -point has continuous cofinal maps, we present the currently known canonical cofinal maps and their implications for the structure of the Tukey types of ultrafilters. This spans several works of Dobrinen, work of Dobrinen and Trujillo, and work of Raghavan.

Our presentation will include the following results: Every monotone cofinal map on an ultrafilter Tukey reducible to a  $p$ -point is continuous (below

some member of the ultrafilter). Every monotone cofinal map on a countable iterate of Fubini products of  $p$ -points is continuous (on some filter base), with respect to the natural topology on the associated tree space. Moreover, every monotone cofinal map on an ultrafilter Tukey reducible to a countable iterate of Fubini products of  $p$ -points is represented by a monotone finitary map. This, combined with a result of Raghavan, relate Tukey reducibility to Rudin-Keisler reducibility. In another vein, we find canonical forms of monotone cofinal maps for ultrafilters associated with a large class of topological Ramsey spaces. These maps are represented by finitary initial-segment preserving maps. Such maps are essential in finding initial Tukey structures. We point out that there are many non- $p$ -points which have this form of canonical cofinal map.

### **Non-rectifiable Delone sets in amenable groups**

Tullia Dymarz

*University of Wisconsin Madison*

`dymarz@math.wisc.edu`

Coauthors: Andres Navas

In 1998 Burago-Kleiner and McMullen constructed the first examples of coarsely dense and uniformly discrete subsets of  $R^n$  that are not biLipschitz equivalent to the standard lattice  $Z^n$ . Similarly we find such subsets inside the three dimensional solvable Lie group SOL that are not biLipschitz equivalent to any lattice in SOL. The techniques involve combining ideas from Burago-Kleiner with quasi-isometric rigidity results from geometric group theory.

### **Some problems in the theory of hyperspaces**

Rodrigo Hernandez-Gutierrez

*University of North Carolina at Charlotte*

`rgutier4@uncc.edu`

Coauthors: Paul Szeptycki

In this talk I would like to speak about some problems in the theory of hyperspaces that I have worked on.

The first problem is about the existence of Whitney maps in the Vietoris hyperspace of continua (compact, connected and Hausdorff space). It is known that metrizable continua admit Whitney maps onto the unit interval but recently it has been asked which non-metrizable continua admit a Whitney map. I will explain why an example constructed by M. Smith admits a Whitney map.

The second of the problems is about normality of the Wijsman hyperspace of a metric space. We do have some partial results and it seems reasonable that this hyperspace is normal only when the base space is separable. However, the general case still eludes us.

### **Barycentric straightening and bounded cohomology**

Jean-Francois Lafont  
*Ohio State University*  
jlafont@math.ohio-state.edu  
Coauthors: Shi Wang (OSU)

I will discuss the barycentric straightening of simplices in irreducible symmetric spaces of non-compact type. If the dimension of the simplex is at least thrice the rank, we will see that the Jacobian of a straightened simplex is uniformly bounded above. Finally, I will explain how this can be used to find bounded representatives for cohomology classes. This was joint work with Shi Wang (OSU).

### **Approaching solenoids and Knaster continua by rays**

Piotr Minc  
*Auburn University*  
mincpio@auburn.edu

I will talk about the following two theorems.

**Theorem 1.** If  $Y$  is a continuum containing a solenoid  $\Sigma$  such that  $R = Y \setminus \Sigma$  is homeomorphic to  $[0, \infty)$ , then, there exists a retraction  $r$  of  $Y$  to  $\Sigma$ . Moreover, any two such retractions are homotopic. It follows that if  $P_R$  is the arc component of  $\Sigma$  containing  $r(R)$ , then  $P_R$  is invariant under every homeomorphism of  $Y$  into itself.

**Theorem 2.** Suppose  $Y$  is a continuum containing a Knaster continuum  $K$  such that  $R = Y \setminus K$  is homeomorphic to  $[0, \infty)$  and  $\text{cl}(R) = Y$ . Let  $e$  denote the endpoint of  $R$  and let  $x$  be an arbitrary point in  $K$ . Then there exists a retraction  $r$  of  $Y$  to  $K$  such that  $r(e) = x$ .

### **Contractibility in hyperspaces**

Patricia Pellicer-Covarrubias  
*National Autonomous University of Mexico*  
paty@ciencias.unam.mx  
Coauthors: Luis paredes-Rivas

Given a continuum  $X$  we denote by  $2^X$  the hyperspace of nonempty compact subsets of  $X$  and by  $C_n(X)$  the subspace of  $2^X$  whose elements have at most  $n$  components.

In some sense Whitney maps measure "the size" of compact subsets of a continuum  $X$ , and they have shown to be a very useful tool in the study of  $2^X$ . A *Whitney level* is a nondegenerate fiber of a Whitney map. It is well known that Whitney levels for  $C(X)$  are continua, and that this is not necessarily the case for Whitney levels of  $C_n(X)$  with  $n \geq 1$ .

Recently H. Hosokawa introduced a generalization of Whitney maps for  $C_n(X)$ , which he called *strong size maps*, with the nice property that their fibers are continua, and he called such fibers *strong size levels*. Furthermore, he defined a topological property  $P$  to be a *strong size property* provided that if a continuum  $X$  has property  $P$ , then so does every strong size level.

In this talk we discuss some properties related to contractibility which are (or fail to be) strong size properties.

### **Three-manifolds with many flat planes.**

Ben Schmidt

*Michigan State University*

`schmidt@math.msu.edu`

Coauthors: Renato Bettiol

A Riemannian manifold is said to have higher rank when each of its geodesics admits a normal, parallel Jacobi field. I'll discuss the proof of the the following rigidity theorem: a complete three manifold has higher rank if and only if its Riemannian universal covering is isometric to a product. Based on joint work with Renato Bettiol.

### **Azumaya algebras and hyperbolic 3-manifolds**

Matthew Stover

*Temple University*

`mstover@temple.edu`

Coauthors: Ted Chinburg and Alan W. Reid

Given a finitely generated group  $G$  and a curve  $C$  in the  $SL(2, C)$  character variety of  $G$ , I will describe how one obtains a so-called Azumaya algebra, a sheaf of quaternion algebras,  $A$  over a Zariski-open subset of  $C$ . When  $G$  is the fundamental group of a cusped hyperbolic 3-manifold  $M$ ,  $C$  is the canonical component containing the discrete and faithful representation, and  $A$  extends over all of  $C$ , this puts significant restrictions on arithmetic invariants of Dehn surgeries on  $M$ . When  $M$  is the complement of a hyperbolic knot  $K$  in  $S^3$ , we can describe a sufficient condition for  $A$  to extend over  $C$  in terms of the Alexander polynomial of  $K$ . For example,  $A$  extends over  $C$  when  $K$  is the figure-eight knot.

## Negativity of Lyapunov exponents of generic random dynamical systems of complex polynomials

Hiroki Sumi

*Osaka University, Japan*

sumi@math.sci.osaka-u.ac.jp

In this talk, we consider random dynamical systems of complex polynomial maps on the Riemann sphere. It is well-known that for each rational map  $f$  of degree two or more, the Julia set is a non-empty perfect compact set (thus it contains uncountably many points), the dynamics of  $f$  on the Julia set is chaotic (at least in the sense of Devaney), and for the set  $A$  of initial points  $z$  in the Riemann sphere at which the Lyapunov exponent is positive, the Hausdorff dimension of  $A$  is positive. Thus the Hausdorff dimension of the chaotic part of the dynamics of  $f$  is positive and there exist uncountably many initial points at which the Lyapunov exponents are positive. However, we show that for a generic i.i.d. random dynamical system of complex polynomials, all of the following (1) and (2) holds.

(1) For all but countably many initial points  $z$  in the Riemann sphere, for almost every sequence of polynomials, the Lyapunov exponent along the sequence starting with  $z$  is negative.

(2) For all points  $z$  in the Riemann sphere, the orbit of the Dirac measure at  $z$  under the dual of the transition operator of the system converges to a periodic cycle of probability measures on the Riemann sphere.

Thus for a generic i.i.d. random dynamical system of complex polynomials, the chaotic part of the averaged system disappears (except countably many points) and the averaged system is much milder than usual dynamical system of a single rational map of degree two or more.

Note that each of (1) and (2) cannot hold in the usual iteration dynamics of a single rational map  $f$  of degree two or more. Therefore the picture of the random complex dynamics is completely different from that of the usual complex dynamics. This is the effect of the randomness and the automatic cooperation (interaction) of many kinds of maps in a random dynamical system. This is called the “cooperation principle”. We remark that even the chaos of the random dynamical system disappears in  $C^0$  sense, the chaos of the system may remain in  $C^1$  sense, and we have to consider the “gradation between the chaos and the order”.

References:

[1] H. Sumi, Random complex dynamics and semigroups of holomorphic maps, Proc. Lond. Math. Soc. (3) 102 (2011), no. 1, 50-112.

[2] H. Sumi, Cooperation principle, stability and bifurcation in random complex dynamics, Adv. Math. 245 (2013), 137-181.

## Cell Structures

E. D. Tymchatyn

*University of Saskatchewan*

tymchat@math.usask.ca

Coauthors: Wojciech Debski

A graph consists of a discrete set of vertices together with a reflexive and symmetric relation on the set of vertices which describes the edges. A cell structure is an inverse system of graphs with bonding maps which satisfy mild continuity and convergence conditions. We show that cell structures suffice to obtain a class of Tychonov spaces which includes compact Hausdorff spaces and complete metric spaces as perfect images of 0-dimensional spaces. Continuous maps between spaces are obtained from cell maps between cell structures. We regard cell structures as providing a kind of bridge between discrete and continuous mathematics. Historically, spaces and maps were obtained using inverse systems of polyhedra and later by Mardesic's resolutions and approximate resolutions. However, in many important cases it is impossible to obtain spaces or their maps with those methods using commutative diagrams. One is forced to use instead diagrams that are only approximately commutative. This makes use of inverse systems and resolutions difficult. Because we use only 1- dimensional data from covers of spaces commutative diagrams suffice for cell structures and cell maps. We expect this work to be of real interest to computer scientists and to people doing topological data analysis.

# Continuum Theory

## Hereditarily irreducible maps between continua

Hussam Abobaker

*Missouri S & T*

haq3f@mst.edu

Coauthors: Włodzimierz J. Charatonik

A surjective continuous function  $f : X \rightarrow Y$  between continua  $X$  and  $Y$  is said to be hereditarily irreducible provided that for any two subcontinua  $A$  and  $B$  of  $X$  such that  $A$  is a proper subcontinuum of  $B$ , the image  $f(A)$  is a proper subcontinuum of  $f(B)$ . We study hereditarily irreducible maps of continua with special attention given to maps between graphs. We introduce a notation of an order of a point, and we show that this order cannot be decreased by a hereditarily irreducible map. Some other invariants of hereditarily irreducible maps are also presented.

## Making holes in the product of some continua

Jose G. Anaya

*Universidad Autonoma del Estado de Mexico*

jgao@uaemex.mx

Coauthors: Enrique Castaneda-Alvarado, Roman Aguirre-Perez, and Pablo Mendez-Villalobos

A connected topological space  $Z$  is *unicoherent*, provided that  $A \cap B$  is connected whenever  $A$  and  $B$  are connected closed subsets of  $Z$  such that  $X = A \cup B$ . A point  $z$  in a unicoherent topological space  $Z$  *makes a hole in*  $Z$  if  $Z - \{z\}$  is not unicoherent. In this talk we will analyze what points makes a hole in the product of either two dendrites, two smooth fans or two Elsa continua.

## Some problems on T-closed subsets of continua

David P. Bellamy

*University of Delaware*

bellamy@udel.edu

Coauthors: Leobardo Fernandez and Sergio Macias

In a compact metric continuum, or for that matter one that is merely Hausdorff, a subset  $A$  is T-closed provided that  $T(A) = A$ . The structure and behaviour of these sets is sometimes surprising. We have made some significant progress on understanding them, in an upcoming article.

In this talk I will discuss some unsolved problems involving such sets.

## More on constructions of R.H. Bing's pseudo-circle in surface dynamics

Jan P. Boroński

*IT4 Innovations, Ostrava*

jan.boronski@osu.cz

Coauthors: Piotr Oprocha

In 1951 R.H. Bing constructed a pseudo-circle, the unique hereditarily indecomposable circle-like cofrontier. The pseudocircle, a fractal-like object, often makes its appearances as an attractor in dynamical systems. Motivated by the results in [1], we study circle maps  $f$  that give the pseudo-circle as the inverse limit space  $\lim_{\leftarrow} \{S^1, f\}$ . We show that any such map exhibits the following properties: (1) there exists an entropy set for  $f$  with infinite topological entropy; i.e.  $h(f) = \infty$ ; (2) the rotation set  $\rho(f)$  is a nondegenerate interval. This shows that the Anosov-Katok type constructions of the pseudo-circle as a minimal set in volume-preserving smooth dynamical systems, or in complex dynamics, obtained previously by Handel, Herman and Chéritat cannot be modeled on inverse limits, and relates to a result of M. Barge concerning certain Hénon-type attractors.

[1] Boronski J.P.; Oprocha P., Rotational chaos and strange attractors on the 2-torus, *Mathematische Zeitschrift* (2015) Vol. 279, Issue 3-4, pp 689-702

## The pseudoarc is a co-existentially closed continuum

Christopher Eagle

*University of Toronto*

cjeagle@math.toronto.edu

Coauthors: Isaac Goldbring and Alessandro Vignati

The main goal of this talk will be to show how compacta can be studied model-theoretically by using continuous first-order logic to study their rings of continuous complex-valued functions. We will compare this approach to other methods of applying model theory in topology, and explain why continuous logic is the natural logic for studying  $C(X)$ . As an application, we will describe our recent result that the pseudoarc is a co-existentially closed continuum, which answers a question of P. Bankston.



### **Idempotency of function $T$ .**

Rocio Leonel Gomez

*Universidad Autonoma del Estado de Hidalgo*

rocioleonel@gmail.com

Coauthors: Carlos Islas

Given a continuum  $X$ , the set function  $T$  was defined by F. B. Jones as follows: if  $A \subset X$ , then  $T(A) = X \setminus \{x \in X : \text{there exists a subcontinuum } W \text{ of } X \text{ such that } x \in \text{Int}(W) \subset W \subset X \setminus A\}$ . We say that  $T$  is idempotent if  $T(T(A)) = T(A)$  for every subset  $A$  of  $X$  and that  $T$  is idempotent on closed sets of  $X$  if  $T(T(A)) = T(A)$  for every closed subset  $A$  of  $X$ . In this talk we present some results on idempotency of  $T$  on closed subsets of  $X$ .

This is a joint work with Carlos Islas.

### **A complete classification of homogeneous plane continua**

Logan Hoehn

*Nipissing University*

loganh@nipissingu.ca

Coauthors: Lex G. Oversteegen

I will discuss some of the methods from our recent proof that the only non-degenerate homogeneous continua in the plane are the circle, the pseudo-arc, and the circle of pseudo-arcs.

The key technical ingredient is the proof that a hereditarily indecomposable continuum with span zero is arc-like, hence homeomorphic to the pseudo-arc. This result follows from a careful analysis of sets in the product of a graph  $G$  and the interval  $[0, 1]$  which separate  $G \times \{0\}$  from  $G \times \{1\}$ . I will discuss such sets in detail, and explain their connection to hereditarily indecomposable continua with span zero.

### **Kelley compactifications of a ray in generalized inverse limits.**

Carlos Islas

*Universidad Autonoma de la Ciudad de Mexico*

carlos.islas@uacm.edu.mx

Coauthors: Isabel Puga

We present a partial answer of a problem of Ingram with respect to compactifications of a ray giving conditions in order that a compactification of a ray to have the property of Kelley

### **On the decomposition of Cartesian product into prime factors**

Daria Michalik

*Institute of Mathematics Polish Academy of Sciences*

d.michalik@uksw.edu.pl

The decomposition of Cartesian product into prime factors is in general not unique. Most of positive results concerning the uniqueness of decomposition deal with 1 dimensional factors. There will be given some sufficient conditions on factors for the uniqueness of decomposition to hold without any dimension restrictions.

### **The topology of continua that admit mixing homeomorphisms**

Christopher G. Mouron

*Rhodes College, Memphis, TN 38112*

mouronc@rhodes.edu

Coauthors: Jorge Martínez and Veronica Martínez de-la-Vega

A map  $f : X \rightarrow X$  is *mixing* if for every open sets  $U, V$  of  $X^n$ , there exists an  $M$  such that  $f^m(U) \cap V \neq \emptyset$  for all  $m \geq M$ . A continuum is *indecomposable* if every proper subcontinuum has empty interior. We will discuss conditions that force continua that admit mixing homeomorphisms to be indecomposable. Also, we will look at an example of a continuum that is hereditarily decomposable but still admits mixing homeomorphisms.

### **Special cases of inverse limits in economics**

Tamalika Mukherjee

*Rochester Institute of Technology*

txm1809@rit.edu

Coauthors: Likin C. Simon Romero

We will introduce the cash in advance model in economics and the use of inverse limits to study the nature of the solutions obtained from the model. We extend the work done J. Kennedy, J. A. Yorke and D. Stockman in their paper, "Inverse limits and an implicitly defined difference equation from economics" by addressing the two cases left open by the authors.

## **A two pass condition for inverse limits with a single set valued bonding map on an interval**

Van Nall

*University of Richmond*

`vnall@richmond.edu`

Coauthors: Judy Kennedy

Let  $f$  be set valued function from  $[0, 1]$  into the closed subsets of  $[0, 1]$ . If the inverse limit with this single bonding map contains two disjoint continua whose first projections are both  $[0, 1]$  then the inverse limit satisfies our two pass condition. We use this notion to explore chaos in the dynamics of set valued functions.

## **The set of buried points of a continuum.**

Norberto Ordonez

*Universidad Autonoma del Estado de Mexico*

`nordonezr@uaemex.com`

Coauthors: Veronica Martinez-de-la-Vega and Fernando Orozco-Zitli

Given a metric continuum  $X$  and a closed subset  $A$  of  $X$ ,  $\mathcal{F}(A)$  denotes the components of  $X - A$ . We define the **set of buried points of  $A$**  as

$$B(A) = A - \cup\{Bd(F) : F \in \mathcal{F}(A)\}$$

Where  $Bd(F)$  denotes the border of  $F$ . In other words,  $B(A)$  is the set of all  $x \in A$  such that  $x$  does not belong to the closure of any component of  $X - A$ . In this talk we will discuss some general properties of the set  $B(A)$  and also we will classify some topological properties of a continuum using the structure of the set  $B(A)$  when  $A$  belongs to  $C(X)$ , where  $C(X)$  is the hyperspace of subcontinua of  $X$  endowed with the Hausdorff Metric.

## **Induced mappings between quotient spaces of $n$ -fold hyperspaces of continua**

Fernando Orozco-Zitli

*Universidad Autonoma del Estado de Mexico, Facultad de Ciencias, Instituto Literario No. 100, Col. Centro, C. P. 50000, Toluca, Estado de Mexico, Mexico.*

`forozcozitli@gmail.com`

Coauthors: Miguel Angel Lara-Mejia and Félix Capulín-Pérez

Let  $X$  be a continuum and let  $n$  be a positive integer. The  $n$ -fold hyperspace  $C_n(X)$  is the set of all nonempty closed subsets of  $X$  with at most  $n$  components. For a mapping  $f : X \rightarrow Y$  between continua, let  $f_n : C_n(X) \rightarrow C_n(Y)$  the induced mapping by  $f$ , defined by  $f_n(A) = f(A)$ . Given  $m$  a positive integer such that  $m < n$ , consider the quotient space  $C_n(X)/C_m(X)$  obtained by shrinking  $C_m(X)$  to a point in  $C_n(X)$ . Let  $\mathcal{F}^{n,m} : C_n(X)/C_m(X) \rightarrow C_n(Y)/C_m(Y)$

the induced mapping by  $f$  given by  $\mathcal{F}^{n,m}(\rho_X^{n,m}(A)) = \rho_Y^{n,m}(f(A))$ , where  $\rho_X^{n,m} : C_n(X) \rightarrow C_n(X)/C_m(X)$  and  $\rho_Y^{n,m} : C_n(Y) \rightarrow C_n(Y)/C_m(Y)$  are the quotient mappings. In this talk we will present some relationships between the mappings  $f$ ,  $f_n$  and  $\mathcal{F}^{n,m}$  for the following classes of mappings: almost monotone, atomic, atriodic, confluent, hereditarily weakly confluent, joining, light, local homeomorphism, locally confluent, locally weakly confluent, monotone, open, OM, semi-confluent, strongly monotone, weakly confluent.

### Selectibility and confluent mappings between fans

Felix Capulin Perez

*Universidad Autonoma del Estado de Mexico*

fcapulin@gmail.com

Coauthors: Leonardo Juarez Villa, Fernando Orozco Zitli

Let  $X$  be a continuum. The hyperspace of all subcontinua of  $X$  is denoted by  $C(X)$ , equipped with the Hausdorff metric. By a *selection* for  $C(X)$  we mean a mapping  $s : C(X) \rightarrow X$  such that  $s(A) \in A$  for each  $A \in C(X)$ .  $X$  is said to be *selectible* provided that there is a selection for  $C(X)$ .

A mapping  $f : X \rightarrow Y$  between continua, is said to be *confluent* provided that for each subcontinuum  $B$  of  $Y$  and each component  $C$  of  $f^{-1}(B)$ , we have  $f(C) = B$ . A *dendroid* is a hereditarily unicoherent and arcwise connected continuum. A *fan* means a dendroid having exactly one ramification point.

In this talk, I am going to present some results about the following question, asked by J. J. Charatonik, W. J. Charatonik and S. Miklos (*Confluent mappings of fans*, *Dissertationes Math.*, **301** (1990), 1-86.) : What kind of confluent mappings preserve selectibility (nonselectibility) between fans?

### Open diameters on graphs

Robert Roe

*Missouri University of Science & Technology*

rroe@mst.edu

Coauthors: Włodzimierz J. Charatonik and Ismail Ugur Tiryaki

We prove that every connected finite graph  $G$  admits a metric for which the diameter function  $diam : 2^G \rightarrow [0, \infty)$  is open, preliminary report.

### **Inverse limits with bonding functions whose graphs are connected**

Şahika Şahan

*Missouri S & T*

ssxx4@mst.edu

Coauthors: Włodzimierz J. Charatonik

First, answering a question by Roškarič and Tratnik, we present inverse sequences of simple triods or simple closed curves with set-valued bonding functions whose graphs are arcs and the limits are  $n$ -point sets. Second, we present a wide class of zero-dimensional spaces that can be obtained as the inverse limits of arcs with one set-valued function whose graph is an arc.

### **Extensions of convex metrics**

Ihor Stasyuk

*Nipissing University*

ihors@nipissingu.ca

Coauthors: Edward D. Tymchatyn

Bing had proved that if  $X_1$  and  $X_2$  are two intersecting Peano continua with convex metrics  $d_1$  and  $d_2$  respectively, there is a convex metric  $d_3$  on  $X_1 \cup X_2$  that preserves  $d_1$  on  $X_1$ . We consider the problem of simultaneous extension of continuous convex metrics defined on subcontinua of a Peano continuum. We construct an extension operator for convex metrics which is continuous with respect to the uniform topology.

### **Chain transitivity of the induced maps $F_n(f)$ and $2^f$**

Artico Ramirez Urrutia

*Universidad Nacional Autonoma de Mexico, Instituto de Matematicas*

articops@gmail.com

Coauthors: Leobardo Fernandez, Chris Good and Mate Puljiz

Let  $X$  be a compact metric space,  $f: X \rightarrow X$  a continuous map and  $\epsilon > 0$ . An  $\epsilon$ -chain is a finite sequence  $\langle x_0, x_1, \dots, x_k \rangle$  such that  $d(f(x_i), x_{i+1}) < \epsilon$  for all  $i \in \{0, 1, \dots, k-1\}$ . In this case, we say the finite sequence is an  $\epsilon$ -chain from  $x_0$  to  $x_k$ . These  $\epsilon$ -chains are also known as (finite)  $\epsilon$ -pseudo-orbits. The map  $f$  is said to be *chain transitive* if for any  $\epsilon > 0$  and  $a, b \in X$ , there exists an  $\epsilon$ -chain from  $a$  to  $b$ .

Roughly speaking, we could say that the chain transitivity is a chain version of the transitivity. Other concepts related to the transitivity can be defined in chain versions. In this talk we will see that the chain transitivity of the induced maps  $F_n(f)$  and  $2^f$  imply many chain properties of the map  $f$ .

## Generalized inverse limits indexed by ordinals

Scott Varagona

*The University of Montevallo*

svaragona@montevallo.edu

The study of generalized inverse limits has become increasingly important to continuum theorists over the last ten years. Many results have been discovered about inverse limits whose factor spaces are indexed by the natural numbers, but we have only begun to scratch the surface of inverse limits whose factor spaces are indexed by other kinds of directed sets. In this talk, we examine generalized inverse limits indexed by certain totally ordered sets, especially ordinals. Various theorems, examples, and problems will be presented.

## Dendrites and Inverse Limits

Paula Ivon Vidal-Escobar

*Universidad Nacional Autonoma de Mexico*

ivon@matem.unam.mx

A very important question in the theory of Inverse Limits of closed subsets of  $[0, 1]^2$  is the following: What spaces can be obtained as an inverse limit of a closed subset of  $[0, 1]^2$ ?

A. Illanes proved that a simple closed curve is not the inverse limit of any closed subset of  $[0, 1]^2$ .

V. Nall proved that the arc is the only finite graph that can be obtained as an inverse limit of a closed subset of  $[0, 1]^2$ .

In this talk I show that a dendrite with a finite number of ramification points is an inverse limit of a closed subset of  $[0, 1]^2$ , if there is an arc in the dendrite containing all of its ramification points and each ramification point has infinite order.

## Embedding cones over graphs into symmetric products

Hugo Villanueva

*Universidad Autonoma de Chiapas, Mexico.*

hugo.villanueva@unach.mx

Coauthors: Florencio Corona and Russell Aaron Quinones (Universidad Autonoma de Chiapas)

For a metric continuum  $X$  and a positive integer  $n$ , let  $F_n$  be the hyperspace of all nonempty subsets of  $X$  with at most  $n$  points, endowed with the Hausdorff metric, and  $Cone(X)$  the topological cone of  $X$ . We say that a continuum  $X$  is cone-embeddable in  $F_n(X)$  provided that there is an embedding  $h$  from  $Cone(X)$  into  $F_n(X)$  such that  $h(x, 0) = x$  for each  $x$  in  $X$ .

In this talk, we present results concerning continua  $X$ , mainly finite graphs, that are cone-embeddable in  $F_n(X)$ .

### **Symmetric Product Graphs**

Evan Witz

*Rochester Institute of Technology*

`emw9004@g.rit.edu`

Coauthors: Likin Simon Romero, Jobby Jacob

In this talk, we will define two new hyperspace graphs (the simultaneous and non-simultaneous symmetric product graphs). We will discuss different properties of these graphs in a graph theoretic sense. A direct connection can be made from these graphs to symmetric product hyperspaces.

## Dynamical Systems

### **Realizing a Non Singular Smale Flow with a Four Band Template**

Kamal Mani Adhikari

*Southern Illinois University, Carbondale*

kadhikari@siu.edu

Coauthors: Michael C Sullivan

The talk will be based on how to realize a Nonsingular Smale Flow using a four band template on 3-manifold. This is an extension of the work done by M. Sullivan about the realization of Lorenz Smale Flow on 3-manifold, work done by Bin Yu about realizing Lorenz Like Smale flows on 3 manifold and the work of Haynes and Sullivan about realizing a Smale Flow using a universal four band template on  $S^3$ .

### **Bowen's formula for non-autonomous graph directed Markov systems**

Jason Atnip

*University of North Texas*

jasonatnip@my.unt.edu

Coauthors: Mariusz Urbanski

We introduce the notion of non-autonomous graph directed Markov systems as well as present a Bowen's formula type result for systems with natural, mild assumptions.

### **An extension of the notion of hypercyclicity to N-linear maps**

Juan Bes

*Bowling Green State University*

jbes@bgsu.edu

Coauthors: J. A. Conejero (Univ. Politecnica de Valencia)

The main object of linear dynamics is the study of hypercyclic operators, that is, continuous linear self-maps of a topological vector space which have a dense orbit. We consider an extension of this phenomena to N-linear operators.



## **Complexity, periodicity, and directional entropy in two dimensions**

Ryan Broderick

*University of California, Irvine*

`broderir@math.uci.edu`

Coauthors: Van Cyr (Bucknell University) Bryna Kra (Northwestern University)

Given a coloring of  $\mathbb{Z}^d$  by a finite color set, one can study the orbit closure under translations as a subshift. The Morse-Hedlund Theorem states that in dimension one, periodicity of this system is equivalent to a condition on its local complexity. M. Nivat conjectured an analogous result in dimension two. We will discuss joint work with V. Cyr and B. Kra in which we show that, given a sequence of complexity assumptions in dimension two, the system is either periodic or has zero directional entropy in all directions.

## **Diophantine extremality and dynamically defined measures**

Tushar Das

*University of Wisconsin – La Crosse*

`tdas@uwlax.edu`

Coauthors: Lior Fishman (North Texas), David Simmons (Ohio State) and Mariusz Urbański (North Texas).

We define a geometric condition which is sufficient for the Diophantine extremality of a measure on Euclidean  $n$ -space, thus generalizing the notion of "friendliness" considered by Kleinbock, Lindenstrauss, and Weiss ('04). This condition is very general and is satisfied by many classes of dynamically defined measures, including all exact dimensional measures of sufficiently large dimension, Patterson–Sullivan measures of nonplanar geometrically finite groups, and conformal measures of infinite IFSes. We will also give some examples of non-extremal measures coming from dynamics, illustrating where the theory must halt. These results are joint-work with Lior Fishman (North Texas), David Simmons (Ohio State) and Mariusz Urbanski (North Texas).

**Hausdorff dimensions of very well intrinsically approximable subsets of quadratic hypersurfaces**

Lior Fishman

*University of North Texas*

lfishman@unt.edu

Coauthors: K. Merrill and D. Simmons

We shall present an analogue of a theorem of A. Pollington and S. Velani ('05), furnishing an upper bound on the Hausdorff dimension of certain subsets of the set of very well intrinsically approximable points on a quadratic hypersurface. The proof incorporates ideas from the work of D. Kleinbock, E. Lindenstrauss, and B. Weiss ('04).

**Plaque inverse limit of a dynamical system - dynamics, signatures and local topology**

Avraham Goldstein

*BMCC/CUNY*

avraham.goldstein.nyc@gmail.com

We will introduce plaque inverse limits of branched covering self-maps of simply-connected Riemann surfaces. We will define the notions of regular and irregular points. At a regular point, a plaque inverse limit has a natural Riemann surface structure, which was studied in the literature since 1990s. However, its local structure at the irregular points was not previously known and is of a great interest. We will introduce certain algebraic machinery, which will permit us to define new local invariants of plaque inverse limits. We call these invariants signatures. The signatures are closely related to the dynamics of the iterations. We will give a description of the local topology of a plaque inverse limit at the irregular points via the signatures. Next, we will present several examples which are of interest in Holomorphic dynamics. Namely, we will consider the invariant lifts of super-attracting, attracting, parabolic and Cremer cycles, the invariant lift of boundaries of rotation domains, and certain irregular point, related to infinitely renormalizable maps with a priori bounds. Using the dynamical properties, we will construct irregular points for these examples and compute their signatures. We will conclude with a stronger version of Mane's theorem.

## Maps with memory

Pawel Gora

*Concordia University , Montreal*

`pawel.gora@concordia.ca`

Coauthors: A. Boyarsky, P. Eslami, Zh. Li and H. Proppe

Let  $f : X \rightarrow X$  be a map. We want to consider a process, which is not a map, and represents situation when  $f$  on each step uses not only current information but also some information from the past. We define for current state  $x_n$  and  $0 < \alpha < 1$ :

$$x_{n+1} = f(\alpha x_n + (1 - \alpha)x_{n-1}).$$

We are interested in something we could call an "invariant measure" of the process. We consider ergodic averages

$$A_f(x_0, x_{-1}) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} f(x_i).$$

They are related to ergodic averages of the map  $G : X \times X \rightarrow X \times X$  defined by

$$G(x, y) = (y, f(\alpha y + (1 - \alpha)x)).$$

We considered the example where  $f : [0, 1] \rightarrow [0, 1]$  is the tent map. Computer experiments suggest that  $G$  behaves in very different manners depending on  $\alpha$ . We conjecture:

For  $0 < \alpha < 1/2$  map  $G$  preserves absolutely continuous invariant measure.

For  $\alpha = 1/2$  every point of upper half of the square ( $y + x \geq 1$ ) has period 3 (except the fixed point  $(2/3, 2/3)$ ). Every other point (except  $(0, 0)$ ) eventually enters the upper triangle.

For  $1/2 < \alpha < 3/4$  point  $2/3, 2/3$  is a global attractor for map  $G$ .

For  $\alpha = 3/4$  every point of the interval  $x + y = 4/3$  has period 2 (except the fixed point  $(2/3, 2/3)$ ). Every other point (except  $(0, 0)$ ) is attracted to this interval.

For  $3/4 < \alpha < 1$  map  $G$  preserves an SRB measure which is not absolutely continuous (supported on an uncountable union of straight intervals).

## The geometry and properties of generalized rotation sets

Tamara Kucherenko

*City University New York, City College*

`tkucherenko@ccny.cuny.edu`

Coauthors: Christian Wolf

For a continuous map  $f$  on a compact metric space we study the geometry and properties of the generalized rotation set of an  $m$ -dimensional continuous potential  $\Phi$ . The generalized rotation set of  $\Phi$  is the set of all  $\mu$ -integrals of  $\Phi$  where  $\mu$  runs over all  $f$ -invariant probability measures. It is easy to see that

the rotation set is a compact and convex subset of  $\mathbb{R}^m$ . We study the question if every compact and convex set is attained as a rotation set of a continuous potential within a particular class of dynamical systems. We give a positive answer in the case of subshifts of finite type by constructing for every compact and convex set  $K$  in  $\mathbb{R}^m$  a potential  $\Phi = \Phi(K)$  which has  $K$  as its rotation set. We also study the relation between the generalized rotation set of  $\Phi$  and the set of all statistical limits of  $\Phi$ . We show that in general these sets differ but also provide criteria that guarantee their equality.

### **The Hilbert-Smith conjecture and the Menger curve**

James Maissen

*University of Texas at Brownsville*

[jmaissen@yahoo.com](mailto:jmaissen@yahoo.com)

Coauthors: James E. Keesling, David C. Wilson

A brief version of the extensive history of this generalization of Hilbert's 5th problem will be given. Historic actions on the Menger curve by the p-adic integers will be described followed by a construction of a new action on it seeing the Menger curve as made from p-adic solenoids. Group actions on Hilbert space will be introduced and an embedding of this action within the free p-adic action on Hilbert space will be given.

### **Identity Return Triangles for Angle-Tripling Map on Laminations of the Unit Disk**

John C. Mayer

*University of Alabama at Birmingham*

[jcmayer@uab.edu](mailto:jcmayer@uab.edu)

Coauthors: Brandon Barry

Quadratic laminations of the unit disk were introduced by Thurston as a vehicle for understanding the Julia sets of quadratic polynomials and the parameter space of (connected) quadratic polynomials, the Mandelbrot set. The "Central Strip Lemma" plays a key role in Thurston's classification of gaps in quadratic laminations, and in describing the corresponding parameter space. In an earlier paper, the speaker and others generalized the notion of Central Strip to laminations of all degrees  $d \geq 2$  and proved a Generalized Central Strip Lemma for degree  $d \geq 2$ . In this talk, we apply the Generalized Central Strip Lemma to analyze the behavior of finite gaps (polygons) under  $\sigma_3$ , the angle-tripling map on the unit circle. In particular, we study *identity return triangles* for  $\sigma_3$ -invariant laminations. The results apply to (non-Cremer) periodic branch points, that return without rotation, in connected cubic polynomial Julia sets. We suggest how these results may be generalized to  $\sigma_d$  for  $d > 3$ .

## Subsets of the cubic connectedness locus, I

Lex Oversteegen

*UAB*

overstee@uab.edu

Coauthors: A. Blokh, R. Ptacek and V. Timorin

The Mandelbrot set  $\mathcal{M}_2$  is the subset of the parameter space of all polynomials of the form  $z^2 + c$  which have a connected Julia set. The cubic connectedness locus  $\mathcal{M}_3$  can be defined similarly. In this talk we study subsets of this space, in particular the subset which is analogous to the Main Cardioid in  $\mathcal{M}_2$ .

## Mapping class semigroups

Kevin M. Pilgrim

*Indiana University*

pilgrim@indiana.edu

Recent work on classification problems arising in one-dimensional complex analytic dynamics suggest an underlying theory of mapping class semigroups. Let  $S^2$  denote the two-sphere, and fix a finite set  $P \subset S^2$ . The set  $BrCov(S^2, P)$  of orientation-preserving branched covering maps of pairs  $f : (S^2, P) \rightarrow (S^2, P)$  of degree at least two and whose branch values lie in  $P$  is closed under composition and under pre- and post-composition by orientation-preserving homeomorphisms  $h : (S^2, P) \rightarrow (S^2, P)$  fixing  $P$  set wise. Composition descends to a well-defined map on homotopy classes relative to  $P$ , yielding a countable semigroup  $BrMod(S^2, P)$ . In addition to the semigroup structure,  $BrMod(S^2, P)$  is naturally equipped with two commuting actions of the mapping class group  $Mod(S^2, P)$ , induced by pre- and post-composition. This richer *biset* structure, and a related circle of constructions, turn out to be extraordinarily useful in this context. They lead to: algebraic invariants of elements of  $BrCov(S^2, P)$  and an analog of the Baer-Dehn-Nielsen theorem; analogs of classical Hurwitz classes; conjugacy invariants; canonical decompositions and forms; an analog of Thurston's trichotomous classification of mapping classes; and induced dynamics on Teichmüller spaces.

This talk is based on algebraic and dynamical perspectives growing out of work of L. Bartholdi and V. Nekrashevych, and is based on ongoing conversations with S. Koch, D. Margalit, and N. Selinger.

## **The Banach-Mazur-Schmidt Game and Banach-Mazur-McMullen Game**

Vanessa Reams

*University of North Texas*

vanessareams@my.unt.edu

Coauthors: L. Fishman, D. Simmons

We introduce two new mathematical games, the Banach-Mazur-Schmidt game and the Banach-Mazur-McMullen game, merging well-known games. We investigate the properties of the games, as well as providing an application to Diophantine approximation theory, analyzing the geometric structure of certain Diophantine sets.

## **Continuity of the Hausdorff measure of infinite iterated function systems**

James Reid

*University of North Texas*

jamesreid2@my.unt.edu

Coauthors: Mariusz Urbanski

Given an infinite conformal iterated function system, we consider its limit set  $J$  and the limit sets  $J_n$  given by the first  $n$  functions. In this talk, we discuss the question, 'when does the limit of the Hausdorff measure of  $J_n$  approach the Hausdorff measure of  $J$ '? We give a characterization in the case of similarities in the unit interval, which leads to some natural examples.

## **Rauzy Tiles of Infimax Systems**

William Severa

*University of Florida*

wsevera@ufl.edu

Infimax sequences are defined as the infimum of all cyclically maximal sequences whose symbols appear in a given proportion. These sequences have been shown to arise as fixed-points under an infinite composition of substitutions, and so they form an S-adic family. Each fixed-point sequence has a geometric representation, called the ladder, and we project this ladder onto a one-dimensional stable manifold. We show that the projection decomposes into Rauzy Tiles which are disjoint intervals with abutting endpoints, and that the shift operator acts on this projection via an interval translation map. The attractor of the dynamical system in the projection is semiconjugate to the orbit closure of the given fixed point.

## **Geometric (re)definitions of Patterson–Sullivan measures**

David Simmons

*Ohio State University*

simmons.465@osu.edu

Given a geometrically finite Kleinian group  $G$ , two naturally associated objects are the limit set  $\Lambda$  and the Patterson–Sullivan measure  $\mu$ . In this talk we discuss the question of whether or not  $\mu$  can be “defined in terms of”  $\Lambda$ . The answer turns out to depend on the Poincaré exponent  $\delta$  and the extremal cusp ranks  $k_{\min}$  and  $k_{\max}$ ; namely, if  $\delta$  is strictly between  $k_{\min}$  and  $k_{\max}$ , then  $\mu$  cannot be defined in terms of  $\Lambda$  via a Hausdorff or packing measure construction based on a gauge function.

## **Realizing Full n-shifts in Simple Smale Flows**

Michael Sullivan

*Southern Illinois University - Carbondale*

mikesullivan@math.siu.edu

Smale flows on 3-manifolds can have invariant saddle sets that are suspensions of shifts of finite type. We look at Smale flows with chain recurrent sets consisting of an attracting closed orbit  $a$ , a repelling closed orbit  $r$  and a saddle set that is a suspension of a full  $n$ -shift and draw some conclusions about the knotting and linking of  $a$  and  $r$ . For example, we show for all values of  $n$  it is possible for  $a$  and  $r$  to be unknots. For any even value of  $n$  it is possible for  $a$  and  $r$  to be the Hopf link, a trefoil and meridian, or a figure-8 knot and meridian.

## **Title: Random Shifts of finite type with weakly positive transfer operator**

Mariusz Urbanski

*University of North Texas*

urbanski.math@unt.edu

Coauthors: Volker Mayer

Abstract: Countable alphabet random subshifts of finite type under the absence of Big Images Property and under the absence of uniform positivity of the transfer operator will be considered. First, the existence of random conformal measures will be established. Then, using the technique of positive cones and a version of Bowen’s type contraction argument, a fairly complete thermodynamical formalism will be built. This means the existence and uniqueness of fiberwise invariant measures (giving rise to a global invariant measure) equivalent to the fiberwise conformal measures. Furthermore, establish the existence of a spectral

gap for the transfer operators. This latter property in a relatively straightforward way entails the exponential decay of correlations and the Central Limit Theorem.

### **Continuity of Hausdorff Dimension**

Tim Wilson

*University of North Texas*

`timothywilson@my.unt.edu`

Coauthors: Mariusz Urbanski

We consider a family of cubic polynomials with attracting fixed point at the origin. We show that continuity of Hausdorff Dimension of the respective Julia sets holds.

### **Localized equilibrium states and rotation sets**

Christian Wolf

*The City College of New York*

`cwolf@ccny.cuny.edu`

Coauthors: Tamara Kucherenko

In this talk we discuss localized equilibrium states for continuous maps on compact metric spaces. These are invariant measures that maximize free energy among those invariant measures whose integral with respect to a given  $m$ -dimensional potential  $\Phi$  has a fixed value  $w$  in the rotation set of  $\Phi$ . It turns out that that even in the case of subshifts of finite type and Hölder continuous potentials, there are several new phenomena that do not occur in the theory of classical equilibrium states. In particular, we construct an example with infinitely many ergodic localized equilibrium states. We also show that for systems with strong thermodynamic properties there is a large class of  $w$ -values with least one and at most finitely many localized equilibrium states.



# Geometric Group Theory

## Conjugacy representatives and hyperbolicity

Yago Antolin

*Vanderbilt University*

yago.anpi@gmail.com

Coauthors: Laura Ciobanu

Rivin conjectured that the conjugacy growth series of a hyperbolic group is rational if and only if the group is virtually cyclic. I will show that the conjugacy growth series of a non-elementary hyperbolic group is transcendental and use this result to prove that in a finitely generated acylindrically hyperbolic group no language containing exactly one minimal length representative of each conjugacy class is regular.

## Bounds on $k$ -systems from the mapping class group

Tarik Aougab

*Yale University*

tarik.aougab@yale.edu

A  $k$ -system on an orientable surface is a collection of pairwise non-homotopic, simple closed curves, pairwise intersecting at most  $k$  times. Using the geometry of the mapping class group, we improve upon the best known upper bounds for the size of such a curve system, and we show that our bounds are close to sharp. Using similar techniques, we also prove that for sufficiently large  $k$ , a large  $k$ -system must have diameter at most 2 in the curve complex.

## Finite rigid sets and homological non-triviality in the curve complex

Nathan Broaddus

*Ohio State University*

broaddus@math.osu.edu

Coauthors: Joan Birman, William Menasco.

Aramayona and Leininger have provided a “finite rigid subset”  $X(S)$  of the curve complex  $C(S)$  of a surface  $S$ , characterized by the fact that any simplicial injection  $X(S) \rightarrow C(S)$  is induced by a unique simplicial automorphism  $C(S) \rightarrow C(S)$ . We prove that, in the case of the sphere with  $n > 4$  marked points, the reduced homology class of the finite rigid set of Aramayona and Leininger is a  $\text{Mod}(S)$ -module generator for the reduced homology of the curve complex  $C(S)$ , answering in the affirmative a question posed by Aramayona and Leininger. For the surface  $S$  with genus  $g > 2$  and  $n = 0$  or  $n = 1$  marked points we find that the finite rigid set  $X(S)$  of Aramayona and Leininger contains a proper subcomplex

whose reduced homology class is a  $\text{Mod}(S)$ -module generator for the reduced homology of  $C(S)$  but which is not itself rigid.

### **Commensurability in right-angled Coxeter groups**

Pallavi Dani

*Louisiana State University*

`pdani@math.lsu.edu`

Coauthors: Emily Stark, Anne Thomas

Commensurability is an algebraic equivalence for groups which implies quasi-isometry. Two groups are abstractly commensurable if they have isomorphic finite index subgroups. I will talk about the commensurability classification of right-angled Coxeter groups based on generalized theta graphs. This extends a result of Crisp and Paoluzzi, and is joint work with Emily Stark and Anne Thomas.

### **On the geometry of the flip graph**

Valentina Disarlo

*Indiana University*

`vdisarlo@indiana.edu`

Coauthors: Hugo Parlier

The flip graph of an orientable punctured surface is the graph whose vertices are the ideal triangulations of the surface and whose vertices are joined by an edge if the two corresponding triangulations differ by a flip. The combinatorics of this graph is crucial in works of Thurston and Penner's decorated Teichmüller theory. In this talk we will explore some geometric properties of this graph, in particular we will see that it provides a coarse model of the mapping class group in which the mapping class groups of the subsurfaces are convex. Moreover, we will provide upper and lower bounds on the growth of the diameter of the flip graph modulo the mapping class group. Joint work with Hugo Parlier (Université de Fribourg).

## Groups with context-free co-word problem

Daniel Farley

*Miami University*

farleyds@miamioh.edu

Coauthors: Rose Berns-Zieve, Dana Fry, Johnny Gillings, Hannah Hoganson, Heather Mathews

Let  $G$  be a group, and let  $S \subseteq G$  be a monoid generating set. Let  $\phi : F(S) \rightarrow G$  be the canonical map from the free monoid on  $S$  to  $G$ . The collection of words that project to the identity in  $G$  is called the *word problem* of  $G$ ; the complement of the word problem in  $F(S)$  is called the *co-word problem*. Both the word problem and the co-word problem are therefore formal languages from this point of view, which raises the possibility of describing the complexity of these problems via the complexity of languages.

A group has a regular word problem (equivalently, a regular co-word problem) if and only if it is finite. Groups with context-free word problem were classified by Muller and Schupp. They are precisely the finitely generated virtually free groups. The class of groups having context-free co-word problem is unknown, but a current conjecture says that a group has context-free co-word problem (i.e., is “coCF”) exactly if it is a finitely generated subgroup of Thompson’s group  $V$ .

I will discuss a couple of classes of groups with context-free co-word problem, one of which arose in an REU at Miami University in 2014. Specifically, I will show that finite similarity structure groups are coCF and that certain groups arising from the “cloning systems” of Witzel and Zaremsky are coCF. Neither class of groups is known to embed in  $V$ , although the conjecture is still open.

## New Examples of Groups with Property (FC)

Talia Fernos

*University of North Carolina at Greensboro*

A group  $\Gamma$  is said to have Property (FC) if any action on a finite dimensional CAT(0) cube complex  $X$  has a fixed point. New non-trivial examples arise by combining the super-rigidity results of Chatterji-Fernós-Iozzi with a description of the structure of point stabilizers in the Roller boundary, established by Caprace. In particular, any irreducible lattice in  $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R})$  has property (FC). In this talk we will discuss the techniques used to prove this result.

## **Snowflake geometry in CAT(0) groups**

Max Forester

*University of Oklahoma*

`forester@math.ou.edu`

Coauthors: Noel Brady

We construct CAT(0) groups containing subgroups whose Dehn functions are given by  $x^s$ , for a large set of exponents  $s \in [2, \infty)$  which includes all rational numbers in this range. This significantly expands the known geometric behavior of subgroups of CAT(0) groups.

## **Closed orbits for quasigeodesic flows**

Steven Frankel

*Yale*

`steven.frankel@yale.edu`

A flow on a manifold is called quasigeodesic if each orbit is coarsely comparable to a geodesic. A flow is called pseudo-Anosov if it has a transverse attracting-repelling structure. If the ambient manifold is hyperbolic, these two conditions are mysteriously related. In fact, Calegari conjectures that every quasigeodesic flow on a closed hyperbolic manifold can be deformed into a pseudo-Anosov flow.

The transverse attracting-repelling structure of a pseudo-Anosov flow lends its orbits a form of rigidity. For example, the Anosov closing lemma says that if a point returns close to itself after a long time, then there's a nearby point that returns exactly to itself.

The goal of this talk is clarify the relationship between quasigeodesic and pseudo-Anosov flows. We will show that a quasigeodesic flow has a \*coarse\* transverse attracting-repelling structure, and use this to prove a closing lemma. In particular, we will show that every quasigeodesic flow on a closed hyperbolic manifold has periodic orbits, answering a question of Calegari's.

## **Constructing Fully irreducible Automorphisms of the Free Group via geometric Dehn twisting**

Funda Gultepe

*UIUC*

`fgultepe@illinois.edu`

Inspired by Thurston's construction of pseudo Anosov diffeomorphisms of the surface using Dehn twists, we construct fully irreducible outer automorphisms of the free group, which are the analogs of pseudo Anosovs in the free group setting. For the construction, we use a notion of a geometric Dehn twist

in the 3 manifold model and the hyperbolicity of the curve complex analogs for the action of the group of outer automorphisms.

**The primitivity index function for a free group, and untangling closed curves on surfaces**

Neha Gupta

*University of Illinois Urbana-Champaign*

ngupta10@illinois.edu

Coauthors: Ilya Kapovich

A theorem of Scott shows that any closed geodesic on a surface lifts to an embedded loop in a finite cover. Our motivation is to find a worst-case lower bound for the degree of this cover, in terms of the length of the original loop. We establish, via probabilistic methods, lower bounds for certain analogous functions, like the Primitivity Index Function and the Simplicity Index Function, in a free group. These lower bounds, when applied in a suitable way to the surface case, give us some lower bounds for our motivating question. This is joint work with Ilya Kapovich.

**On the geometry of  $k$ -free hyperbolic 3-manifold groups**

Rosemary Guzman

*University of Iowa*

rosemary-guzman@uiowa.edu

In the 1990s, Culler, Shalen, and their co-authors initiated a program to understand the relationship between the topology and geometry of a closed hyperbolic 3-manifold. In this talk, we extend those results to the setting of hyperbolic 3-manifolds with the property that all subgroups of  $\pi_1(M)$  of rank at most  $k$  are free (a readily discernible property described as  $k$ -free), discuss the proof and implications. Antecedents of this work, including work of Anderson, Canary, Culler and Shalen in the 3-free case and Culler and Shalen in the 4-free case, used geometric information about a selected point  $P$  to deduce lower bounds on the volume of  $M$  deriving that  $\text{vol}(M) \geq 3.44$ . We will touch on possible applications to improving this bound.

### **Local topology of CAT(0) boundaries**

Chris Hruska

*University of Wisconsin-Milwaukee*

`chruska@uwm.edu`

Coauthors: Kim Ruane

It is known, by a theorem of Swarup, that the boundary at infinity of any word hyperbolic group is locally connected. But when is the boundary of a CAT(0) group locally connected? I will give a complete solution to this question in the setting of CAT(0) spaces with isolated flats. In this setting the boundary is an invariant of the group (not depending on which space the group acts on). The solution uses Bowditch's notion of peripheral splittings of a relatively hyperbolic group.

### **Abelian splittings of Right-Angled Artin groups.**

Michael Hull

*University of Illinois at Chicago*

`mbhull@uic.edu`

Coauthors: Daniel Groves

We will discuss when (and how) a Right-Angled Artin group splits nontrivially over an abelian subgroup.

### **Rigidity in coarse hyperbolic geometry**

Kyle Kinneberg

*Rice University*

`kk43@rice.edu`

Rigidity phenomena have been of great interest in Riemannian geometry in the past few decades, especially in the context of negative curvature. Motivated by rigidity results in this setting, we explore analogues for Gromov hyperbolic groups. In particular, we will discuss a coarse entropy-rigidity theorem and related questions about rigidity through curve families on the boundary at infinity.

## **Finiteness property of subgroups of the symmetric group on $\mathbb{Z}$**

Sang Rae Lee

*Texas*

srlee@math.ou.edu

Consider elements of the symmetric group  $S$  on  $\mathbb{Z}$  which are eventual periodic maps. A group  $G_p$  which consists of eventual periodic maps with period  $p$  has a finite index subgroup  $G$  fitting into

$$1 \rightarrow N \rightarrow G \rightarrow \mathbb{Z}^m \rightarrow 1$$

where  $N$  is a torsion subgroup of  $S$ . In this talk we study finiteness property of groups of eventual periodic maps. We show each group in this class has type  $F_{n-1}$  but not  $F_n$  for some positive integer  $n$ . For the proof we construct CAT(0) cubical complexes on which those groups acts. We also show that this class contains Houghton's groups.

## **Can an orbifold be isospectral to a manifold?**

Benjamin Linowitz

*University of Michigan*

linowitz@umich.edu

An old problem asks whether a Riemannian manifold can be isospectral to a Riemannian orbifold with nontrivial singular set. We show that under the assumption of Schanuel's conjecture in transcendental number theory, this is impossible whenever the orbifold and manifold in question are length commensurable compact locally symmetric spaces of nonpositive curvature associated to simple Lie groups.

## **Large Scale Absolute Extensors and Asymptotically Lipschitz functions**

Atish Mitra

*Montana Tech (University of Montana)*

atish.mitra@gmail.com

Coauthors: J Dydak

In an earlier paper the author (with Jerzy Dydak) defined the concept of large scale absolute extensors of metric spaces, and related it to asymptotic dimension of Mikhail Gromov. We will talk about subsequent work relating large scale absolute extensors with extensions of proper, asymptotically Lipschitz functions. We will discuss applications to coarse geometry and geometric group theory.

## **Between subexponential asymptotic dimension growth and Yu’s Property A**

Izhar Oppenheim  
*The Ohio State University*  
oppenheim.10@osu.edu

Property A is a quasi isometric invariant that was introduced by Yu as a “non equivariant” version of amenability in order to show that groups with this property coarsely embed into a Hilbert space. Property A is weaker than other quasi isometric invariants connected to Gromov’s asymptotic dimension. Specifically, recently it was shown by Ozawa that property A is implied by subexponential asymptotic dimension growth (it is unknown if the two are equivalent). In my talk, I’ll present a new idea of proving this implication based on a new quasi isometric invariant called asymptotically large depth.

## **Separability Properties of Right-Angled Artin Groups**

Priyam Patel  
*Purdue University*  
pate1376@purdue.edu  
Coauthors: Khalid Bou-Rabee, Mark Hagen

Right-Angled Artin groups (RAAGs) and their separability properties played an important role in the recent resolutions of some outstanding conjectures in low-dimensional topology and geometry. We begin this talk by defining two separability properties of RAAGs, residual finiteness and subgroup separability, and provide a topological reformulation of each. We then discuss joint work with K. Bou-Rabee and M.F. Hagen regarding quantifications of these properties for RAAGs and the implications of our results for the class of virtually special groups.

## **Geometry of the conjugacy problem**

Andrew Sale  
*Vanderbilt University*  
andrew.sale@vanderbilt.edu  
Coauthors: Yago Antolin

The classic conjugacy problem of Max Dehn asks whether, for a given group, there is an algorithm that decides whether pairs of elements are conjugate. Related to this is the following question: given two conjugate elements  $u, v$ , what is the shortest length element  $w$  such that  $uw = vw$ ? The conjugacy length function (CLF) formalises this question. I will survey what is known for CLFs of groups. I will also discuss a new closely related function, the permutation conjugacy length function (PCL), outline its potential application to studying



the computational complexity of the conjugacy problem, and describe a result, joint with Y. Antolin, for the PCL of relatively hyperbolic groups.

### **Random Groups at Density 1/2**

Andrew P. Sanchez

*Tufts University*

`andrew.sanchez@tufts.edu`

Coauthors: Moon Duchin, Kasia Jankiewicz, Shelby C. Kilmer, Samuel Lelievre, and John M. Mackay

In the density model of random groups, a set  $R$  of relators is chosen (uniformly) randomly from freely reduced words of length  $\ell$  in  $m$  generators, and we send  $\ell \rightarrow \infty$ . Let us define the (*generalized*) *density*  $d = \lim_{\ell \rightarrow \infty} (\log_{2m-1} |R|) / \ell$ .

The classic theorem in this model is that for  $d < 1/2$ , random groups are (asymptotically almost surely) infinite hyperbolic, while for  $d > 1/2$ , random groups are (asymptotically almost surely) trivial.

Gromov left open the behavior at the threshold density  $d = 1/2$ . In my joint work with Duchin, Jankiewicz, Kilmer, Lelievre, and Mackay, we have found that both hyperbolic and trivial groups can be found at  $d = 1/2$ . (This extends and generalizes work by Kozma.)

### **Different notions of discreteness and the modified Poincare exponent**

David Simmons

*Ohio State University*

`simmons.465@osu.edu`

In this talk I will discuss various definitions of discreteness for a group acting isometrically on a metric space. I will introduce the *modified Poincare exponent* of such a group and discuss its geometric interpretation in the case that the metric space is hyperbolic. Finally, I will describe the relation between the modified Poincare exponent, the usual Poincare exponent, and our different notions of discreteness. This work is joint with Tushar Das and Mariusz Urbanski.

## **Algebraic degrees and Galois conjugates of pseudo-Anosov stretch factors**

Balazs Strenner

*UW-Madison*

`strenner@math.wisc.edu`

Coauthors: Hyunshik Shin

Thurston showed that the algebraic degrees of stretch factors of pseudo-Anosov maps on the orientable surface of genus  $g$  are bounded by  $6g-6$ . He claimed that his Dehn twist construction produces examples of maximal  $6g-6$  degree stretch factors, but he did not give a proof. I will discuss a method based on a construction of Penner that produces maximal degree examples, and also examples of all even degrees less than  $6g-6$ . I will also mention related work with Hyunshik Shin, where we study Galois conjugates of stretch factors, and use this to resolve a conjecture of Penner.

## **The Geometry of Random Right-angled Coxeter Groups**

Tim Susse

*University of Nebraska - Lincoln*

`tsusse2@unl.edu`

Coauthors: Jason Behrstock, Mark Hagen, Victor Falgas-Ravry

We will begin by presenting a model, based on the Erdos-Renyi random graph model, of producing random right-angled Coxeter groups. We will discuss two strong connectedness properties of the generating graph and give thresholds at which these properties are generic. We will then describe strong connections between the structure of the generating graph and the geometry of the corresponding group due to Dani-Thomas and Behrstock-Hagen-Sisto. Together, these imply that at a wide range of densities, a random right-angled Coxeter group almost surely has quadratic divergence.

## **Tits Rigidity of CAT(0) group boundaries**

Eric Swenson

*Brigham Young University*

`eric@math.byu.edu`

We define Tits rigidity for visual boundaries of CAT(0) groups, and prove that the join of two Cantor sets and its suspension are Tits rigid.

## Shadows of Teichmueller Discs in the curve graph

Robert Tang

*University of Oklahoma*

rtang@math.ou.edu

Coauthors: Richard Webb

Given a flat structure on a surface, we can deform the metric using elements of  $SL(2, R)$  to obtain different metrics. The  $SL(2, R)$ -orbit under this action is called a Teichmueller disc. We consider several sets of curves naturally associated with a Teichmueller disc: its systole set, its cylinder set and its set of bounded weight curves. We show that these sets are quasiconvex and agree up to bounded Hausdorff distance in the curve graph of the surface. We also describe nearest point projections to these sets.

## DIVERGENCE OF MORSE GEODESICS

Hung Tran

*University of Wisconsin-Milwaukee*

hctran@uwm.edu

Behrstock and Drutu raised a question about the existence of Morse geodesics in  $CAT(0)$  spaces with divergence function strictly greater than  $r^n$  and strictly less than  $r^{n+1}$ , where  $n$  is an integer greater than 1. In this paper, we answer the question of Behrstock and Drutu by showing that there is a  $CAT(0)$  space  $X$  with a proper, cocompact action of some finitely generated group such that for each  $s$  in  $(2, 3)$  there is a Morse geodesic in  $X$  with divergence function equivalent to  $r^s$ .

## Sigma-invariants of generalized Thompson groups

Matt Zaremsky

*Binghamton University*

zaremsky@math.binghamton.edu

Coauthors: Stefan Witzel

The so called Sigma-invariants  $\Sigma^m(G)$  of a group  $G$  (with  $m$  ranging over the naturals) are a catalog describing precisely how highly connected certain filtrations by half spaces are, in spaces associated to the group. In 2010, Bieri, Geoghegan and Kochloukova computed all the  $\Sigma^m(F)$  for Thompson's group  $F$ . In 2012 Kochloukova further computed  $\Sigma^2(F_n)$  for a family of generalized Thompson groups  $F_n$  (of which  $F$  is  $F_2$ ). In recent joint work with Stefan Witzel, we recovered the  $\Sigma^m(F)$ , via an entirely geometric proof using a  $CAT(0)$  cube complex on which  $F$  acts, and I was subsequently able to compute  $\Sigma^m(F_n)$  for all  $m$  and  $n$ , extending Kochloukova's  $m = 2$  result, using the action of  $F_n$  on a  $CAT(0)$  cube complex. In this talk I will discuss the groups and the  $CAT(0)$

cube complexes, state the results, and give an idea of why certain half spaces are connected and others are not.

# Geometric Topology

## **1-systems and their dual cube complexes**

Tarik Aougab

*Yale University*

tarik.aougab@yale.edu

Coauthors: Jonah Gaster

We study dual cube complexes associated to certain special configurations of curves on surfaces called complete 1-systems. These are precisely the complete subgraphs of the so-called “Schmutz graph”, whose large scale geometry is that of the curve graph, but whose local geometry is not well-understood. By analyzing those cube complexes which are dual to such curve systems, we prove that the number of mapping class group orbits of complete 1-systems of maximum size grows exponentially in the genus of the underlying surface. This is joint work with Jonah Gaster.

## **Convex cocompactness and stability in mapping class groups**

Matthew Durham

*University of Michigan*

durhamma@umich.edu

Coauthors: Samuel Taylor

Originally defined by Farb-Mosher to study hyperbolic extensions of surface subgroups, convex cocompact subgroups of mapping class groups have deep ties to the geometry of Teichmüller space and the curve complex. I will present a strong notion of quasiconvexity in any finitely generated group, called stability, which coincides with convex cocompactness in mapping class groups. This is joint work with Samuel Taylor.

## **Cotorsion-free groups from a topological viewpoint**

Hanspeter Fischer

*Ball State University*

fischer@math.bsu.edu

Coauthors: Katsuya Eda (Waseda University)

We present a characterization of cotorsion-free abelian groups in terms of homomorphisms from fundamental groups of Peano continua, which aligns naturally with the generalization of slenderness to non-abelian groups. In the process, we calculate the first homology group of the Griffiths twin cone.

## **Unitary equivalence classes of matrices over $C(X)$**

Greg Friedman

*Texas Christian University*

`greg.b.friedman@gmail.com`

Coauthors: Efton Park (TCU)

Let  $C(X)$  be the ring of continuous functions on a topological space  $X$ . It is a question of analytic interest to determine when two normal matrices with entries in  $C(X)$  are unitarily equivalent, i.e. when one is the conjugate of another by a unitary matrix. We will show that the only obstruction to such an equivalence is an element in the second cohomology of  $X$  with certain twisted coefficients and derive some consequences of this fact.

## **Length spectral rigidity for strata of Euclidean cone metrics**

Ser-Wei Fu

*Temple University*

`swfu@temple.edu`

When considering Euclidean cone metrics on a surface induced by quadratic differentials, there is a natural stratification by prescribing cone angles. I will describe a simple method to reconstruct the metric locally using the lengths of a finite set of closed curves. However, there is a surprising result that a finite set of simple closed curves cannot be length spectrally rigid when the stratum has enough complexity. I will also sketch a brief history of the length spectral rigidity problem.

## **Finitary maps**

Ross Geoghegan

*Binghamton University (SUNY)*

`ross@math.binghamton.edu`

Here is a general situation:  $G$  is a group,  $S$  and  $T$  are  $G$ -sets, and one is interested in maps from  $S$  to  $T$  which, in some reasonable way, respect the  $G$ -action. The strongest notion is that of  $G$ -map, but it is sometimes the case that there are too few of those, or that they all have some rigidity structure (e.g a fixed point) which the interesting cases do not have. One wants something in between  $G$ -maps and arbitrary maps.

The module version of the same issue is this:  $A$  and  $B$  are  $ZG$ -modules and the nicest maps from  $A$  to  $B$  are  $ZG$ -homomorphisms, while the least nice (worth discussing) are  $Z$ -homomorphisms.

In recent joint work with Robert Bieri we had the need for an in-between category of maps, the  $G$ -finitary category. The objects are finitely generated  $ZG$ -modules and the morphisms “respect the  $G$ -action up to a finite set of choices”.

This sounds (and is) very general - almost a philosophy of how to deal with ZG-modules - but we have real applications in controlled topology/geometric group theory.

In this short talk I'll attempt to explain the idea and indicate how it can be used to have the necessary kind of control in an interesting topological situation.

### **Infinite boundary connected sums and exotic universal covering spaces**

Craig Guilbault

*University of Wisconsin-Milwaukee*

`craig@uwm.edu`

Coauthors: Fredric Ancel

We will revisit a widely-known, but never published, theorem by Siebenmann and Ancel about the topological structure of the exotic universal covers of aspherical  $n$ -manifolds constructed by Mike Davis in the 1980's. The fundamental observation underlying that unpublished work involves the topological structure of manifolds obtained by the process of taking "infinite boundary connected sums". In this talk we will discuss some generalizations of the above, along with several new applications. Among the applications are: a proof that Davis' exotic covering spaces can be expressed as a union of open sets  $U \cup V$ , with  $U$ ,  $V$  and  $U \cap V$  all homeomorphic to  $\mathbb{R}^n$ ; and the construction of a collection of aspherical  $n$ -manifolds with universal covers even more exotic than the original examples.

### **Free transformations of $S^1 \times S^n$ of prime period**

Qayum Khan

*Saint Louis University*

`khanq@slu.edu`

Let  $p$  be an odd prime, and let  $n$  be a positive integer. We classify the set of equivariant homeomorphism classes of free  $C_p$ -actions on the product  $S^1 \times S^n$  of spheres, up to indeterminacy bounded in  $p$ . The description is expressed in terms of number theory.

The techniques are various applications of surgery theory and homotopy theory, and we perform a careful study of  $h$ -cobordisms. The  $p = 2$  case was completed by B Jahren and S Kwasik (2011). The new issues for the odd  $p$  case are the presence of nontrivial ideal class groups and a group of equivariant self-equivalences with quadratic growth in  $p$ . The latter is handled by the composition formula for structure groups of A Ranicki (2009). This paper has been generalized to allow for the period  $p$  to be square-free odd.

### **Arbitrarily long factorizations in mapping class groups**

Mustafa Korkmaz

*Middle East Technical University*

korkmaz@metu.edu.tr

Coauthors: Elif Dalyan and Mehmetcik Pamuk

On a compact oriented surface of genus  $g$  with  $n \geq 1$  boundary components,  $\delta_1, \delta_2, \dots, \delta_n$ , we consider positive factorizations of the boundary multitwist  $t_{\delta_1} t_{\delta_2} \cdots t_{\delta_n}$ , where  $t_{\delta_i}$  is the positive Dehn twist about the boundary  $\delta_i$ . We prove that for  $g \geq 3$ , the boundary multitwist  $t_{\delta_1} t_{\delta_2}$  can be written as a product of arbitrarily large number of positive Dehn twists about nonseparating simple closed curves, extending a recent result of Baykur and Van Horn-Morris, who proved this result for  $g \geq 8$ . This fact has immediate corollaries on the Euler characteristics of the Stein fillings of contact three manifolds.

### **Remarks on stable homotopy categorification**

Igor Kriz

*University of Michigan*

kriz.igor@gmail.com

Coauthors: P. Hu, P. Somberg

I will discuss an analog of the BGG category of  $gl_n$ -representations in modules over the sphere spectrum, and work on progress on possible applications to constructing new stable homotopy versions of categorification.

### **Relatively hyperbolic Coxeter groups of type HM**

Giang Le

*The Ohio State University*

le.145@osu.edu

A Coxeter group is type HM if it has an effective, proper and cocompact action on some contractible manifold. We study relatively hyperbolic Coxeter groups of type HM with flats of codimension 1. In this talk, we will present one of our results which says that the dimension of these groups is bounded above.



### **Constructing effective homotopies**

Fedor Manin

*University of Chicago*

`manin@math.uchicago.edu`

Coauthors: Greg Chambers, Dominic Dotterrer, Shmuel Weinberger

Suppose we have two maps between compact simplicial complexes  $X$  and  $Y$  which are  $L$ -Lipschitz, or simplicial on a certain subdivision, and which we know are homotopic. To what extent can we control the size of a homotopy between them? In the case where  $Y$  is simply connected, Gromov (1999) and Ferry-Weinberger (2013) offer a range of conjectures. In ongoing work, we disprove one such conjecture and offer evidence for others.

### **Totally geodesic spectra and rigidity**

Jeff Meyer

*University of Oklahoma*

`jmeyer@math.ou.edu`

Coauthors: D. B. McReynolds and Matthew Stover

Do the collection of totally geodesic subspaces of an orbifold determine its isometry class? If not, perhaps its commensurability class? In this talk, we will outline recent work over the past couple years addressing these questions for standard arithmetic real and quaternionic hyperbolic orbifolds, and, more generally, for locally symmetric orbifolds of type  $B_n$ ,  $C_n$ , and  $D_n$ . For such spaces, the totally geodesic subspaces indeed determine its commensurability class. Interestingly, it is even often sufficient to restrict to distinguished subclasses of totally geodesic subspaces. For example, the complex hyperbolic subspaces of a quaternionic hyperbolic orbifold determine its commensurability class, but its real hyperbolic subspaces do not. Conversely, in joint work with D. B. McReynolds and Matthew Stover, we construct arbitrarily large families of nonisometric  $2n$ -dimensional ( $n > 2$ ) real hyperbolic orbifolds with the same set of totally geodesic subspaces.

## **Combinatorial Systolic Inequalities**

Barry Minemyer

*Ohio State University*

`minemyer.1@osu.edu`

Coauthors: Ryan Kowalick, Jean-François Lafont

In this talk (research is joint with Ryan Kowalick and J.F. Lafont) I will establish combinatorial versions of various classical systolic inequalities. For a smooth triangulation of a closed smooth manifold, the minimal number of edges in a homotopically non-trivial loop contained in the 1-skeleton gives an integer called the combinatorial systole. The number of top-dimensional simplices in the triangulation gives another integer called the combinatorial volume. Our main theorem is that a class of smooth manifolds satisfies a systolic inequality for all Riemannian metrics if and only if it satisfies a corresponding combinatorial systolic inequality for all smooth triangulations. Along the way, we show that any closed Riemannian manifold has a smooth triangulation which "remembers" the geometry of the Riemannian metric, and conversely, that every smooth triangulation gives rise to Riemannian metrics which encode the combinatorics of the triangulation. I will also show how our main result can be used to "fill" triangulated surfaces via a triangulated 3-manifold with a bounded number of tetrahedra.

## **Metrics on the Visual Boundary of CAT(0) Spaces**

Molly Moran

*University of Wisconsin-Milwaukee*

`mamoran@uwm.edu`

For a visual Gromov hyperbolic space, the asymptotic dimension of the space is at most one more than the linearly controlled dimension of the boundary (Buyalo, 2005). One could ask if a similar result holds for CAT(0) spaces or groups. If so, it would finally settle a long standing open question about finiteness of the asymptotic dimension of CAT(0) groups. However, since linearly controlled dimension is a metric invariant and metrics on the visual boundary of CAT(0) spaces have not been studied in a significant way, we first must develop metrics on the boundary that induce the cone topology. In this talk, we discuss two possible metrics and describe dimension results that we have obtained with these metrics.

### **Nonpositively curved concordances**

Pedro Ontaneda

*Binghamton University (SUNY)*

pedro@math.binghamton.edu

We define nonpositively curved concordances and study this concept in three simple cases: closed manifolds, simply connected manifolds and some products.

### **Flat Strips and Bowen-Margulis Measures for Rank One CAT(0) Spaces**

Russell Ricks

*University of Michigan*

rmricks@umich.edu

For compact, negatively curved manifolds, the measure of maximal entropy provides an important tool for understanding the dynamics of the geodesic flow. This measure, constructed independently by Bowen and Margulis using different methods, is called the Bowen-Margulis measure. Sullivan later provided another method to construct this measure for real hyperbolic space; this method has since been generalized to many geometric settings, including compact, rank one nonpositively curved manifolds and CAT(-1) spaces under proper isometric actions. We will discuss a generalization to the setting of certain rank one CAT(0) spaces and identify precisely which CAT(0) spaces are mixing.

Our construction of the Bowen-Margulis measure hinges on establishing a new structural result of independent interest: Almost no geodesic (under the Bowen-Margulis measure) bounds a flat strip of any positive width. We also show that almost every point in the boundary of  $X$  (under the Patterson-Sullivan measure) is isolated in the Tits metric.

### **Embedding buildings and action dimension**

Kevin Schreve

*University of Wisconsin-Milwaukee*

kevinschreve@gmail.com

The action dimension of a group is the minimal dimension of a contractible manifold that the group acts on properly discontinuously. I will explain how action dimension is connected to the minimal embedding dimension of simplicial complexes in  $\mathbb{R}^n$ . I will also show that spherical buildings are generally hard to embed in  $\mathbb{R}^n$ .

## **The Lusternik-Schnirelmann category of Peano continua**

Tulsi Srinivasan  
*University of Florida*  
tsrinivasan@ufl.edu

We extend the theory of the Lusternik-Schnirelmann category (LS-category) to Peano continua by means of covers by general subsets. We obtain upper bounds for the LS-category of Peano continua by proving analogues to the Grossman-Whitehead theorem and Dranishnikov's theorem, and obtain lower bounds in terms of cup-length, category weight and Bockstein maps. We use these results to calculate the LS-category for some fractal spaces like Menger spaces and Pontryagin surfaces. We compare this definition with Borsuk's shape theoretic LS-category. Although the two definitions do not agree in general, our techniques can be used to find similar upper and lower bounds on the shape theoretic LS-category, and we compute its value on some fractal spaces.

## **Dehn twists and the intersection number on nonorientable surfaces**

Michal Stukow

Following the recent result about two Dehn twists generating a free group on a nonorientable surface I would like to show a couple of interesting differences between orientable and nonorientable surfaces.

## **Homomorphisms from the mapping class group of a nonorientable surface**

Blazej Szepietowski  
*Gdansk University*  
blaszecz@mat.ug.edu.pl

Let  $M(N_g)$  denote the mapping class group of the closed nonorientable surface of genus  $g$ . We look at homomorphisms from  $M(N_g)$  to  $GL(m, C)$  and to mapping class groups of other surfaces. We will show that for  $g > 4$  and  $g > m - 1$  the image of a homomorphism from  $M(N_g)$  to  $GL(m, C)$  is abelian, and thus has order at most 4. The same is true for the image of a homomorphism from  $M(N_g)$  to  $M(N_h)$  if  $g > h$ . For  $g > 6$  we will give a classification of representations of  $M(N_g)$  in  $GL(g - 1, C)$ .

## Random extensions of free groups and surface groups

Samuel Taylor

*Yale*

s.taylor@yale.edu

Coauthors: Giulio Tiozzo

In this talk, I will explain the meaning behind the following theorem, which is joint work with Giulio Tiozzo: A random extension of either a free group (of rank at least 3) or a surface group (of genus at least 2) is Gromov hyperbolic.

## Z-Structures on Baumslag-Solitar Groups

Carrie Tirel

*University of Wisconsin - Fox Valley*

carrie.tirel@uwc.edu

Coauthors: Craig Guilbault, Molly Moran, Christopher Mooney

A  $Z$ -structure on a group  $G$  is a pair  $(X, Z)$  of spaces such that  $X$  is a compact ER,  $Z$  is a  $Z$ -set in  $X$ ,  $G$  acts properly and cocompactly on  $X' = X \setminus Z$ , and the collection of translates of any compact set in  $X$  forms a null sequence in  $X$ . It is natural to ask whether a given group, or class of groups, admits a  $Z$ -structure. The prototypical examples are hyperbolic and  $CAT(0)$  groups, all of which admit  $Z$ -structures. While Bestvina asserts that  $BS(1,2)$  admits a  $Z$ -structure in his paper "Local Homology Properties of Boundaries of Groups", we will show that, in fact, every Baumslag-Solitar group admits a  $Z$ -structure.

## Barycentric straightening and bounded cohomology

Shi Wang

*The Ohio State University*

wang.2187@buckeyemail.osu.edu

Coauthors: Jean Lafont (OSU)

In this talk, I will report on joint work with Jean Lafont. Let  $X = G/K$  be an irreducible symmetric space of non-compact type, and  $\Gamma$  a cocompact torsion-free lattice in  $G$ . We show that the comparison maps  $\eta : H_{c,b}^p(G, \mathbb{R}) \rightarrow H_c^p(G, \mathbb{R})$  and  $\eta' : H_b^p(\Gamma, \mathbb{R}) \rightarrow H^p(\Gamma, \mathbb{R})$  are both surjective in all degrees  $p \geq 3\text{rank}(X)$ . This gives an almost complete answer to a question of Dupont. For the main technique, we use the barycentric straightening of simplicies and show the  $p$ -Jacobian has uniformly bounded norm, when  $p \geq 3\text{rank}(X)$ , this generalizes the Jacobian estimate of Connell and Farb. I will continue on Jean Lafont's talk and give more details on the approach.

## **On Poincare duality for orbifolds**

Dmytro Yeroshkin  
*Syracuse University*  
dyeroshk@syr.edu

This talk will examine the obstructions to integer-valued Poincare duality for (underlying spaces of) orbifolds. In particular, it will be shown that in dimensions 4 and 5, the obstruction is controlled by the orbifold fundamental group. A consequence of this is that if the orbifold fundamental group is naturally isomorphic to the fundamental group of the underlying space, then the orbifold satisfies integer-valued Poincare duality.

## Set-Theoretic Topology

### Some new results on essential ideals in rings of continuous functions

Sudip Kumar Acharyya

*Department of Pure Mathematics, University of Calcutta, 35 Ballygunge Circular Road, Kolkata-700019, West Bengal, India*

sdpacharyya@gmail.com

Coauthors: Pritam Rooj

A non zero ideal  $I$  in a commutative ring  $R$  with identity is called essential if for each  $a \neq 0$  in  $R$ , there exists  $b \neq 0$  in  $R$  such that  $ab \neq 0$  and  $ab \in I$ . It turns out that, if an ideal  $I \neq (0)$  in the ring  $C(X)$  of all real valued continuous functions over a Tychonoff space  $X$  is free in the sense that, there does not exist any point  $x$  in  $X$  for which  $f(x) = 0$  for each  $f \in I$ , then  $I$  is essential in  $C(X)$ . For any ideal  $P$  of closed sets in  $X$ , let  $C_P(X) = \{f \in C(X) : cl_X\{x \in X : f(x) \neq 0\} \in P\}$  and  $C_\infty^P(X) = \{f \in C(X) : \text{for each } \epsilon > 0 \text{ in } \mathbb{R}, cl_X\{x \in X : |f(x)| > \epsilon\} \in P\}$ . Then  $C_P(X) \subseteq C_\infty^P(X)$  but  $C_P(X)$  need not be contained in the ring  $C^*(X)$  of all bounded real valued continuous functions over  $X$ . However if  $f \in C_\infty^P(X)$  and  $g \in C^*(X)$  then  $fg \in C_\infty^P(X)$  from which it follows that,  $C_\infty^P(X)$  is an ideal of the ring  $C_\infty^P(X) + C^*(X)$ . We first show that, the following three statements are equivalent:

1.  $C_P(X)$  is an essential ideal of  $C(X)$
2.  $C_\infty^P(X)$  is an essential ideal of  $C_\infty^P(X) + C^*(X)$ .
3.  $X$  is almost locally  $P$  in the sense that, each nonempty open set  $U$  in  $X$  contains a nonempty open set  $V$  such that,  $cl_X V \subseteq U$  and  $cl_X V \in P$ .

A special case of this proposition on choosing  $P = K \equiv$  the ideal of all compact sets in  $X$  reads as follows:

1.  $C_K(X)$  is an essential ideal of  $C(X)$
2.  $C_\infty(X)$  is an essential ideal of  $C^*(X)$ .
3.  $X$  is almost locally compact in the sense that, each nonempty open set  $U$  in  $X$  contains a nonempty open set  $V$  such that,  $cl_X V \subseteq U$  and  $cl_X V$  is compact.

This last fact is established by Azarpanah in 1997 [A]. Furthermore on choosing  $P \equiv$  the ideal of all closed relatively pseudo compact subsets of  $X$  and using the fact that, for an  $f \in C(X)$ ,  $cl_X\{x \in X : f(x) \neq 0\}$  is pseudo compact if and only if it is a relatively pseudo compact subset of  $X$ , a fact obtained by Mandelkar in 1971 [M], we have recorded a second special case of the above mentioned proposition:

1. The ring  $C_\psi(X)$  of all functions in  $C(X)$  with pseudo compact support is an essential ideal of  $C(X)$ .

2.  $C_\infty^\psi(X) = \{f \in C(X) : \text{for each } \epsilon > 0 \text{ in } \mathbb{R}, \text{ } cl_X\{x \in X : |f(x)| > \epsilon\} \text{ is pseudo compact}\}$  is an essential ideal of  $C^*(X)$ .
3.  $X$  is almost locally pseudo compact in the sense that, each nonempty open set  $U$  in  $X$  contains a nonempty open set  $V$  such that,  $cl_X V \subseteq U$  and  $cl_X V$  is pseudo compact.

Finally the problem when  $C_P(X)$  becomes identical to the intersection of all essential ideals in  $C(X)$  is also addressed.

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### On the category of approach convergence rings

T. M. G. Ahsanullah

*Department of Mathematics, King Saud University, Riyadh 11451, Saudi Arabia*  
 tmga1@ksu.edu.sa

Starting with the convergence approach structure attributed to Lowen et al (cf. [5]), we introduce a category of approach convergence rings **ApConvRing**, and look at some of its related categories. We also introduce the notions of approach uniform group and approach uniform convergence group along with some of their basic facts; and show that every approach convergence ring carries in a natural way an approach uniform convergence structure. Here too, we look at some category related properties of the structures involved. We explore the links between the categories of approach Cauchy rings, **ApChyRing** and approach convergence rings **ApConvRing**, and beyond.

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### **Banach Contraction Principle in 3-variable extension of multiplicative metric spaces with Applications to Linear Algebra and Differential Equations**

Clement Boateng Ampadu

*31 Carrolton Road, Boston, MA, 02132*

[drampadu@hotmail.com](mailto:drampadu@hotmail.com)

Coauthors: None

Multiplicative metric spaces was introduced in [1]. In this talk, we introduce a multiplicative metric in 3-variables and use it to prove Banach Contraction Principle in this setting. We possibly, consider application to Linear Algebra and Differential Equations

References

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Note: This might possibly be a preliminary report

### **Generalized Metrics**

Samer assaf

*University of Saskatchewan -Saskatoon-Canada*

[Samerassaf@hotmail.com](mailto:Samerassaf@hotmail.com)

Coauthors: Koushik Pal

In this talk we explore the generalization of a metric to a: partial metric ( by S. Matthews and O'Neil), D-metric( by Dhage), G-Metric (by Z. Mustafa and B. Sims ) , G<sub>p</sub> - Metric ( by M. Zand and A. Nzehad) and K-metric ( K. Khan). Subsequently,we combine the above notions into one structure called a Partial n-Metric space and study properties of limit and completeness.

## **A Quotient Construction**

Jocelyn Bell

*United States Military Academy at West Point*

bell.jocelyn@gmail.com

In terms of separation properties, quotients of “nice” spaces need not be “nice”. We define a natural equivalence relation on a product of uniform spaces which is an extension of a familiar metric space construction. This equivalence relation imparts a (Hausdorff) uniform structure on the quotient space. This quotient can be nicer still: we show that for a certain non-metrizable space, this construction inherits strong covering and separation properties beyond those guaranteed by the uniform structure.

## **Coreflections and generalized covering space theories**

Jeremy Brazas

*Georgia State University*

jbrazas@gsu.edu

Generalizations and extensions of classical covering space theory appear throughout the literature and have applications in geometric topology, infinite group theory, and topological group theory. Typically, generalized covering maps are defined to retain the unique lifting properties enjoyed by covering maps (in the classical sense). In this talk, I will discuss the use of coreflective categories of topological spaces to construct a unifying categorical framework for covering-like maps defined purely in terms of lifting properties. The framework will be applied to three special coreflective categories: (1) The category of Delta-generated spaces provides a “universal” category of coverings in which all others embed. (2) The category of locally-path connected spaces retains a familiar notion of generalized covering due to Fischer-Zastrow (3) The coreflective hull of the directed arc-fans characterizes the notion of “continuous lifting” used in semicovering theory.

## **Limited information strategies for a topological proximal game**

Steven Clontz

*Auburn, AL*

steven.clontz@gmail.com

The proximal property was introduced by Jocelyn Bell in 2014 to generalize collectionwise normality and countable paracompactness, and was shown by the author and Gary Gruenhagen to characterize Corson compactness among compact spaces. The proximal property was originally characterized by the existence of a winning strategy for the first player in a certain game played on a uniform structure inducing the topology of the space. The author will outline

an analogous purely topological game which also characterizes the proximal property, as well as results related to the existence of limited information ( $k$ -Markov and  $k$ -tactical) strategies in this topological proximal game.

### **Compact, countable tightness, and trees**

Alan Dow  
*UNC Charlotte*  
adow@uncc.edu

We discuss the common connection with trees in understanding the structure of “counter examples” related to some well-known problems about compact spaces of countable tightness.

### **The pseudoarc is a co-existentially closed continuum**

Christopher Eagle  
*University of Toronto*  
cjeagle@math.toronto.edu  
Coauthors: Isaac Goldbring and Alessandro Vignati

The main goal of this talk will be to show how compacta can be studied model-theoretically by using continuous first-order logic to study their rings of continuous complex-valued functions. We will compare this approach to other methods of applying model theory in topology, and explain why continuous logic is the natural logic for studying  $C(X)$ . As an application, we will describe our recent result that the pseudoarc is a co-existentially closed continuum, which answers a question of P. Bankston.

## Collins-Roscoe Structuring Mechanism and D-property

Ziqin Feng

*Auburn University*

zzf0006@auburn.edu

Coauthors: John Porter (Murray State University)

The Collins-Roscoe condition (F) was introduced in the context of metrization and generalized metric spaces. A collection  $\mathcal{W} = \{\mathcal{W}(x), x \in X\}$  satisfies condition (F) if  $y \in U$  and  $U$  is open, then there exists an open set  $V = V(y; U)$  containing  $y$  such that  $x \in V(y; U)$  implies  $y \in W \subseteq U$  for some  $W \in \mathcal{W}(x)$ . Gruenhage first proved that countable (F) spaces are hereditarily D. Xu showed that well-ordered (F) spaces are also hereditarily D. We show that  $\sigma$ -sheltering (F) spaces are hereditarily D.

## Minimum Group Topologies

Paul Gartside

*University of Pittsburgh*

gartside@math.pitt.edu

Coauthors: Xiao Chang

Let  $G$  be a group. A Hausdorff topological group topology,  $\tau$ , on  $G$  is *minimal* if there is no Hausdorff topological group topology on  $G$  strictly contained in  $\tau$ , and is the *minimum* if every Hausdorff topological group topology on  $G$  contains  $\tau$ .

We show that some homeomorphism groups of continua have a minimum group topology, and show that many others have no minimum group topology. In doing so we answer various questions raised by others.

## Cardinalities of weakly Lindelöf spaces with regular $G_\delta$ -diagonals

Ivan S. Gotchev

*Central Connecticut State University*

gotchevi@ccsu.edu

In this talk we will present three upper bounds for the cardinality of a space with a regular diagonal. As direct corollaries of these results we obtain the following: If  $X$  is a space with a regular  $G_\delta$ -diagonal then: (1)  $|X| \leq 2^{wL(X)}$ ; (2)  $|X| \leq wL(X)^{\chi(X)}$ ; and (3)  $|X| \leq aL(X)^\omega$ ; where  $\chi(X)$ ,  $wL(X)$  and  $aL(X)$  are respectively the character, the weak Lindelöf number and the almost Lindelöf number of  $X$ .

It follows from (1) that the cardinality of every space  $X$  with a regular  $G_\delta$ -diagonal does not exceed  $2^{c(X)}$  and that every weakly Lindelöf space with a regular  $G_\delta$ -diagonal has cardinality at most  $2^\omega$ . These generalize R. Buzyakova's result that the cardinality of a ccc-space with a regular  $G_\delta$ -diagonal does not

exceed  $2^\omega$ . It also follows from (1) that if  $X$  is a space with a regular  $G_\delta$ -diagonal then  $|X| \leq 2^{\chi(X) \cdot wL(X)}$ ; hence Bell, Ginsburg and Woods inequality  $|X| \leq 2^{\chi(X)wL(X)}$ , which is known to be true for normal  $T_1$ -spaces, is true also for a large class of spaces that includes all spaces with regular  $G_\delta$ -diagonals.

Inequality (2) improves significantly Bell, Ginsburg and Woods inequality for the class of normal spaces with regular  $G_\delta$ -diagonals. In particular (2) shows that the cardinality of every first countable space with a regular  $G_\delta$ -diagonal does not exceed  $wL(X)^\omega$ .

For the class of spaces with regular  $G_\delta$ -diagonals (3) improves Bella and Cammaroto inequality  $|X| \leq 2^{\chi(X) \cdot aL(X)}$ , which is valid for all Urysohn spaces. Also, it follows from (3) that the cardinality of every almost Lindelöf space with a regular  $G_\delta$ -diagonal does not exceed  $2^\omega$ .

### **Trying to characterize the Complex Stone Weierstrass Property**

Joan Hart

*University of Wisconsin Oshkosh*

hartj@uwosh.edu

Coauthors: Ken Kunen

We look at open questions and recent results related to the Complex Stone–Weierstrass Property. A compact space  $X$  has the Complex Stone–Weierstrass Property (CSWP) iff for all subspaces  $E$  of the space  $C(X, \mathbb{C})$  of continuous functions from  $X$  into the field of complex numbers: If  $E$  is a subalgebra of  $C(X, \mathbb{C})$  that separates points and contains all constant functions, then  $E$  is dense in  $C(X, \mathbb{C})$ . If, in the preceding definition, we replace the complex numbers with the reals, then every compact space  $X$  has the resulting property, by the Stone–Weierstrass Theorem. The CSWP for compact metric spaces was characterized by W. Rudin in the 1950s, and for compact LOTs by K. Kunen in the 2000s. It is still unknown whether, for arbitrary compacta, there is an equivalent to the CSWP in terms of standard notions of topology. We look at recent somewhat scattered attempts to approximate the CSWP.

### **On generalized open sets**

Sabir Hussain

*Qassim University, Saudi Arabia*

sabiriub@yahoo.com

In this talk, the characterizations and properties of newly defined generalized open sets will be presented. Examples and counter examples will be given in favour of claims.

### **Forcing and Frechetness**

Akira Iwasa

*University of South Carolina Beaufort*

iwasa@uscb.edu

We consider the following six mutually disjoint classes of spaces: 1. First countable 2. Strongly Frechet but not first countable 3. Frechet but not strongly Frechet 4. Sequential but not Frechet 5. Countably tight but not sequential 6. Not countably tight We discuss if forcing can make a space in one of the above classes belong to a different class.

### **Networks on free topological groups**

Chuan Liu

*Ohio University - Zanesville*

liuc1@ohio.edu

Coauthors: Fucai Lin

In this talk, I will present some results related with networks on free topological groups over generalized metric spaces. Some questions are posed.

### **Tukey order, ordinals and trees**

Ana Mamatelashvili

*Auburn University*

azm0105@auburn.edu

Tukey ordering compares cofinal complexity of directed partially ordered sets. For two directed posets,  $P$  and  $Q$ ,  $P \geq_T Q$  if and only if there is a map,  $\phi : P \rightarrow Q$ , that maps cofinal subsets of  $P$  to cofinal subsets of  $Q$ . For a space  $X$ , define  $\mathcal{K}(X)$  to be the set of all compact subsets of  $X$  ordered with set inclusion. In this talk we will consider the Tukey ordering on  $\mathcal{K}(X)$ 's, where  $X$  is a tree or a subset of  $\omega_1$ . We will also define a relative Tukey ordering for pairs of posets  $(P', P)$ , where  $P' \subseteq P$ . This allows us to compare pairs of the form  $(X, \mathcal{K}(X))$ .

### **P-proximity spaces**

Joseph Van Name

*CUNY*

`jvanname@mail.usf.edu`

We shall generalize the notion of a  $P$ -space to proximity spaces. The category of all  $P$ -proximity spaces is equivalent to the category of all  $\sigma$ -algebras. Furthermore, the  $P$ -proximity coreflection of a proximity space corresponds to the  $\sigma$ -algebra of all proximally Baire sets.

### **Big fundamental groups: generalizing homotopy and big homotopy**

Keith Penrod

*Morehouse College*

`keith.penrod@morehouse.edu`

Cannon and Conner defined the notion of big fundamental group and showed that for any Hausdorff space, the group is well-defined. We show that the group is well defined for any space and give a canonical big interval which can be used to calculate the big fundamental group. In particular, for any infinite cardinal  $\alpha$ , we have a functor  $\pi_1^\alpha$  and show that for any space  $X$ , the big fundamental group  $\Pi_1(X, x_0)$  as defined by Cannon and Conner is isomorphic to  $\pi_1^\alpha(X, x_0)$  for some canonical  $\alpha$ .

### **Examples of pseudo-compact spaces and their products**

Leonard R. Rubin

*University of Oklahoma*

`LRUBIN@OU.edu`

Coauthors: Ivan Ivansic (University of Zagreb)

A space is called pseudo-compact if every map of it to the real line has compact image. The study of such spaces began during the 1950s. In general, examples of pseudo-compact spaces that are not compact were constructed on a case-by-case basis. A famous example of such is the first uncountable ordinal space. Some models of noncompact pseudo-compacta were constructed to show that this property, unlike that of compactness, is not infinitely or even finitely productive.

Recently we have discovered large classes of noncompact, pseudo-compact spaces that exist “naturally.” We have constructed such spaces as direct limits of inclusion direct systems, some of which were induced by sets of graphs of maps. We have found in this class a large number that satisfy the finite product theorem. Going further, we have also located major classes of such spaces that can be constructed as the direct limits of inclusion direct systems using certain collections of subsets of Tychonoff cubes. We were able to show that for many

of these, pseudo-compactness is preserved under arbitrary products. We will discuss such examples in our presentation.

### **Compactness properties and $\omega$ -continuous maps**

Vladimir V. Tkachuk

*Universidad Autonoma Metropolitana de Mexico*

vova@xanum.uam.mx

Coauthors: O.G. Okunev

This is a joint work with O. Okunev. The operation of extending functions from  $X$  to  $\nu X$  is  $\omega$ -continuous, so it is natural to study  $\omega$ -continuous maps systematically if we want to find out which properties of  $C_p(X)$  "lift" to  $C_p(\nu X)$ . We study the properties preserved by  $\omega$ -continuous maps and bijections both in general spaces and in  $C_p(X)$ .

We show that  $\omega$ -continuous maps preserve primary  $\Sigma$ -property as well as countable compactness. On the other hand, existence of an  $\omega$ -continuous injection of a space  $X$  to a second countable space does not imply  $G_\delta$ -diagonal in  $X$ ; however, existence of such an injection for a countably compact  $X$  implies metrizability of  $X$ . We also establish that  $\omega$ -continuous injections can destroy caliber  $\omega_1$  in pseudocompact spaces.

In the context of relating the properties of  $C_p(X)$  and  $C_p(\nu X)$ , a countably compact subspace of  $C_p(X)$  remains countably compact in the topology of  $C_p(\nu X)$ ; however, compactness, pseudocompactness, Lindelöf property and Lindelöf  $\Sigma$ -property can be destroyed by strengthening the topology of  $C_p(X)$  to obtain the space  $C_p(\nu X)$ . We show that Lindelöf  $\Sigma$ -property of  $C_p(X)$  together with  $\omega_1$  being a caliber of  $C_p(X)$  implies that  $X$  is cosmic.

### **Domain representability implies generalized subcompactness**

Lynne Yengulalp

*University of Dayton*

lyengulalp1@udayton.edu

A regular space  $X$  is called generalized subcompact if there is a base  $B$  for  $X$  and a relation  $<$  defined on  $B$  that satisfies

1.  $<$  is antisymmetric and transitive,
2.  $U < V$  implies  $U \subseteq V$ ,
3. for all  $x$  in  $X$ ,  $\{U : x \in U\}$  is downward directed by  $<$ , and
4. if  $F \subseteq B$  is downward directed by  $<$ , then  $\bigcap F$  is non-empty.

A space is subcompact if there is a base for which the order  $U < V$  iff  $cU \subseteq V$  satisfies the above properties. It is known that generalized subcompactness is strictly weaker than subcompactness.

We prove that for regular spaces, domain representability is equivalent to generalized subcompactness.