

Archimedean ℓ -Groups

WARREN WM. MCGOVERN

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1. Fundamental concepts of ℓ -groups

We will discuss those concepts and objects whose understanding is pivotal in the understanding of lattice-ordered groups. This will include a discussion of torsion freeness, convex ℓ -subgroups, prime subgroups, values (i.e. regular subgroups), group homomorphisms, polar subgroups, etc.

2. Examples of ℓ -groups

We shall give an exhaustive list of lattice-ordered groups.

3. Classes of ℓ -groups

Here we consider those special classes of ℓ -groups that have been important in the literature. We shall discuss and give examples of varieties of ℓ -groups, radical classes of ℓ -groups, (quasi-)torsion classes of ℓ -groups, etc.

4. Properties of ℓ -groups

We shall draw fine distinctions between the different types of ℓ -groups via different properties: Dedekind complete ℓ -groups, (strongly) projectable ℓ -groups, archimedean ℓ -groups, hyper-archimedean ℓ -groups, Specker and hyper- \mathbb{Z} ℓ -groups, etc.

5. Archimedean ℓ -groups

We will discuss the Yosida Representation of an archimedean ℓ -group with weak order unit. We will then discuss several interesting properties of ℓ -groups and their characterizations as \mathbf{W} -objects.

6. Groups of Invertible Ideals

We will discuss how to construct partially ordered groups from a given commutative integral domain. In particular, we will discuss the group of divisibility and the group of invertible ideals. We will explore the connection between the theory of ℓ -groups and domain theory.

7. (Time permitting) Algebraic K -theory

We will discuss the application of the theory of ordered groups to algebraic K -theory.