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Grade Level: Six

Lesson: Probability (2-3 days)

Introduction

There is a certain beauty about mathematics. It is like a deep and mysterious cave that only reveals itself as it is explored. One of the unique properties of elementary probability concepts is that they have an incredible depth. I (Ryan) remember studying probability back in junior high and high school, and I remember thinking that it was fairly simple and not all that exciting. However, it was not until I re-encountered probability in *Discrete Mathematics* and *Probability and Statistics* during my undergraduate work at Azusa Pacific University that I began to appreciate the dynamic beauty of the nature of probabilities.

The student's experience with probability will depend mostly on his or her mathematical maturity or mathematical curiosity. If the teacher and student are content with merely stepping into the cave of probability long enough to learn how to replicate mathematical procedures on test, the experience will probably be dull. However, if the student and teacher take time to explore the cave by grappling with the ideas of probability, I can guarantee that both the teacher and student will walk away enriched mathematically, academically, and intellectually.

The lesson plan was written toward the mathematical maturity of the sixth graders in the school where the authors are working; however, it is the belief of the authors that the plan can be easily simplified or complicated to suit a quite wide range of students. In fact, these basic exercises in experimental probability could expand the average students' understanding of probability for students in lower level undergraduate math classes.

Academic Content Standards: Grade Indicator(s) and Benchmark(s):

Benchmarks:

By the end of the 3-4 program:

- F. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.
- G. Identify and represent possible outcomes, such as arrangements of a set of up to four members and possible combinations from several sets, each containing two or three members.
- H. Use the set of possible outcomes to describe and predict outcomes.

By the end of the 5-7 program:

- E. Collect, organize, display and interpret data for a specific purpose or need.
- G. Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays.
- H. Find all possible outcomes of simple experiments or problem situations, using methods such as lists, arrays and tree diagrams.
- J. Compare experimental and theoretical results for a variety of simple experiments.
- K. Make and justify predictions based on experimental and theoretical probabilities.

Grade Level-Indicators:

Grade 4:

9. Conduct simple probability experiments and draw conclusions from the results; e.g., rolling number cubes or drawing marbles from a bag.
10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.

Grade 5:

7. List and explain all possible outcomes in a given situation.
8. Identify the probability of events within a simple experiment, such as three chances out of eight.
9. Use 0,1 and ratios between 0 and 1 to represent the probability of outcomes for an event, and associate the ratio with the likelihood of the outcome.
10. Compare what should happen (theoretical expected results) with what did happen (experimental/actual results) in a simple experiment.
11. Make prediction based on experimental and theoretical probabilities.

Grade 6:

6. Make logical inferences from statistical data.
7. Design an experiment to test a theoretical probability and explain how the results may vary.

Essential Understandings/Big Ideas:

Find experimental and theoretical probabilities

Write probabilities as a fraction/ratio and then decimals and percent.

“5-E” Phase

Engage: The M&M Probability Game (Estimated Time: 10 min)

Notes: Materials, Graphic Organizers, Cooperative Learning

6 colored marbles (3 red, 2 blue, 1 white)

1 paper bag

At least 5 M&Ms per student

1 4*6 note card per student

Planned Activities/Events:

Each student is given a 4*6 note card. The teacher should then direct the student to divide the note card into five parts and label them as indicated below.

1
2
3
4
5

Next, the teacher is going to ask the students to guess which color marble is going to be drawn from the bag. If the student guesses correctly they will get a M&M. The student can guess either red, blue, or white. The teacher should then display the colored marbles (see engage notes below). The teacher should verbally note that there are 3 red marbles, 2 blue marbles, and 1 white marble. The student is to use a pen (to make sure guesses do not get ‘changed’) to write their first guess in the section of the note card labeled “1.”

The teacher will then place the marbles into a paper bag and shake the paper bag around to randomize the marble drawn. Before drawing a marble, the teacher should ask the students to finalize their guess about the color they think will be drawn. The teacher should then draw a marble from the bag. After each draw the marble should be put back in the bag. Repeat the exercise four more times.

Before passing out the M&Ms, instruct the students to not to eat the M&Ms. Also, ask them to think about their choice of colors and if they would do anything different if the game were to be played again. Next, pass out M&Ms based on the answers given on the answer cards.

Now, make a quick observation of how many M&Ms some of the students have. Now, the teacher should begin a discussion about the nature of what M&Ms the students chose to bet on. Perhaps ask a few students what colors they guessed. Most importantly, the teacher must ask the question “Why did you choose the color you did?” Also, ask, “Would you change your guesses if we played again?” Hopefully, this discussion will refresh the basic ideas of probability for the students.

After this engage, the teacher should be sure that the students can determine basic probabilities such as there is a 1/2 chance of a “heads” to come up on a coin flip or there is a 1/6 of a chance a “4” comes up on the roll of a die. If needed, the class can work together to find the probabilities of drawing each color from the bag.

Guiding Questions

What color did you choose?

Why did you choose the color you did?

Would you change your guesses if we played again?
Why was it important that the marbles were put back in the bag?
Would the colors always be drawn in the same order?
Do you think the same colors would be drawn in we did five more draws?

Explore: Experimental Probability (Estimated Time: 30 min)

Notes: Materials, Graphic Organizers, Cooperative Learning

Flipping Coin Worksheet

Teacher set up: one coin needed per coin flipping station.

Rolling Die Worksheet

Teacher set up: one six-sided die needed for each station

Pulling Marbles I

Teacher set up: one paper bag and marbles (recommended: 1 red, 2 white, 3 blue)

Note: If the teacher would like to make time, an excellent exercise would be to combine the information from all the groups for each station. Combining all their data would increase the sample size, and most likely bring the experimental probabilities closer to the theoretical probabilities. This would be an excellent opportunity to implement cooperative learning.

Planned Activities/Events

During the explore part of the lesson there will be three stations set up. Ideally, the students should be in groups of about three or four; consequently, depending on class size, it might be necessary to set up two of each station. The students should spend about ten minutes at each station.

Before splitting into groups, the students should either know or be reminded of the formulas for experimental and theoretical probabilities. Recall:

Experimental probability = number of outcomes of an event/number of experiments.

Theoretical probability = number of possible outcomes of an event/number of possible outcomes.

Other important terminology that the students should know and begin to use is:

Experiment: one trial such as one flip of a coin or one roll of a dice.

Outcome: the result of an experiment

Guiding Questions

The teacher should circulate the classroom during this time to ensure that all members of the group are on task. The teacher should also try to correct any misunderstanding or misconceptions the students might have.

Explain (Estimated Time: 10 min)

Notes: Materials, Graphic Organizers, Cooperative Learning

Results from the experiments carried out by the class.

Planned Activities/Events

Lead a class discussion about the results from the experiments. Perhaps have the groups compare the experimental probabilities that they found. Most likely, all the experimental probabilities will be different but they should be similar. Perhaps, as part of the “explain”, the teacher could lead a discussion based on the “Note” above. The student should gain an emerging understanding of the correlation between sample size and the accuracy of the experimental probability. The student should also understand what the experimental probability represents and what the theoretical probability represents.

Guiding Questions

Why are all the experimental probabilities different?

If we combine all of the class’s results, are our experimental probabilities closer to the theoretical probabilities? Why?

What is the difference between the experimental probability and the theoretical probability?

Extend: The Mystery Bag (Estimated Time: 30 min)

Notes: Materials, Graphic Organizers, Cooperative Learning

Preparation: The teacher should prepare one mystery bag for each group of students. Each bag should contain about 10 marbles. The 10 marbles should be composed of random amounts of blue, red, and white marbles.

Planned Activities/Events

Each group is given a Mystery Bag. The group is to draw one marble at a time. The group should record the color of each marble and then put the marble back in the bag. This process should be repeated until the students feel they have a reasonable estimate for the number of each color marble in the bag. The only information the students are given is the number of marbles in the bag. The students should use the fact there are 10 marbles in the bag and their experimental probabilities to arrive at an answer.

Guiding Questions

How many “draws” should you do?

Is 5 draws enough? 10? 100?

How did you find your answer?

Explain (Time: 10 min)

Planned Activities/Events

As a class, review how the groups arrived at a conclusion. One good question is to ask how many draws each group used. Then compare that with how accurate their results were.

Guiding Questions

How many draws did you use?

How did you arrive at that number?

Extend (Time: 40 min)

Notes: Materials, Graphic Organizers, Cooperative Learning

Note: This extension is quite advanced for sixth graders. It is quite possible that they do not understand the full significance of the exercise. However, the goal of the exercise is that they be able to realize when a probability is fixed and when it is dynamic. Because of the nature of many gambling games, this is an important concept. Example, the Roulette wheel never changes. Even if 10 reds happen a row, the odds are not greater for a black than they were before.

Preparation: Each group should be given a paper bag with 10 marbles. I recommend making the distribution of red, blue, and white marbles the same for each group just so the results can be discussed. Perhaps 5 red, 3 blue, and 2 white.

Planned Activities/Events

This part of the lesson is composed of two parts; an engage and a group activity. The engage is basically a new version of the M&M probability game that is described in the first engage. However, when the teacher draws a marble from the bag, they should not replace it! The teacher should verbally emphasize the fact that the marble is not being put back in bag.

(*) Each group should look into the bag and determine what the probability is for their first draw. They should record those probabilities on the worksheet. Then, a member of the group should reach into the bag and draw marble. If there is any marbles left in the bag return to (*). Complete the experiment 2 times.

Post-experiment discussion should bring up the idea that the probabilities changes after each draw based on the marbles drawn before.

Guiding Questions

- Compare the two tables.
- Are they the same?
- Are they different?
- Why might they be different?

Evaluate (Time:)

Planned Activities/Events

An excellent worksheet title *Roll 'Em!* can be found in *The Mailbox* from June/July 1993. The worksheet is a good assessment for how advanced the student's comprehension of probability is. It is not an easy/rote worksheet for sixth graders.

Also, there is an included rubric for the student's group work.

Technical Notes

1. Quite obviously, in the M&M Probability Game, any reward or prize may be used. The reason that we used M&Ms is that all the students know that they are one of our favorite candies so it seems to make it more interesting for the kids.

2. The main reason that we chose “marbles” for use in most of the experiments is that they are most common in probability and statistics texts. However, a good alternative is the simple plastic poker chips. The chips come in red, blue, and white and generally do not have anything relating them to poker on them. One distinct advantage of the poker chips is that they do not have the rolling tendency of the marbles.

Appendix

Flipping Coin Worksheet

Rolling Die Worksheet

Pulling Marbles I (with replacement)

Pulling Marbles II (without replacement)

Rubric (for the group work)

Flipping a Coin

The goal of this experiment is to find the “experimental probability” of flipping a coin. Conduct 25 “experiments” by flipping the coin and recording the “outcome” in the boxes below.

What is the experimental probability of flipping a heads?

What is the experimental probability of flipping a tails?

What is the theoretical probability of flipping a heads?

What is the theoretical probability of flipping a tails?

Is the experimental probability for flipping a heads and flipping a tails close to the theoretical probability?

Now we are going to conduct 25 more experiments to see if the experimental probabilities get closer to the theoretical probabilities.

What is your new experimental probability for flipping heads?

What is your new experimental probability for flipping tails?

Are these closer to the theoretical probabilities?

How many heads would you expect to come up for 50 experiments (coin flips) based on the theoretical probability?

How many tails would you expect to come up for 50 experiments (coin flips) based on the theoretical probability?

Why?

Rolling a Die

The goal of this experiment is to find the “experimental probability” of rolling a die. Conduct 40 “experiments” by rolling the die and recording the “outcome” in the boxes below.

Record the experimental and theoretical probabilities of each of the six outcomes in the table below.

	1	2	3	4	5	6
Experimental Probability						
Theoretical Probability						

Are the experimental probabilities close to the theoretical probabilities?

What could we do to make the experimental probabilities closer to the theoretical probabilities? (hint: recall the coin flipping experiments)

What would you expect the results to be for 120 experiments (rolls) based on the theoretical probabilities?

	1	2	3	4	5	6
Number of Outcomes						

Pulling Marbles I

The goal of this experiment is to find the “experimental probability” of pulling certain marbles out of a bag. Conduct 36 “experiments” by pulling a marble out of the bag and recording the “outcome” in the boxes below. Important! Put the marble back in the bag after recording the outcome.

Based on your results above, record the experimental probabilities and theoretical probabilities of each outcome below.
 Note: To determine the theoretical probabilities, you will have to take the marbles out of the bag so that you can see them all.

	red	blue	white
Experimental Probability			
Theoretical Probability			

Are the experimental probabilities close to the theoretical probabilities?
 What could we do to make the experimental probabilities closer to the theoretical probabilities? (hint: recall the coin flipping experiments)

What would you expect the results to be for 120 experiments based on the theoretical probabilities?

	red	blue	white
Number of Outcomes			

Pulling Marbles II

1. Look in the bag and determine the probability of drawing each color.
2. Draw a marble without looking (do not put the marble back in the bag!).
3. If there is any marbles left in the bag, return to step 1 and repeat.

	0	1	2	3	4	5	6	7	8	9
Color Drawn	X									
Red										
Blue										
White										

Repeat the experiment

	0	1	2	3	4	5	6	7	8	9
Color Drawn	X									
Red										
Blue										
White										

Compare the two tables. Are they the same? Are they different? Why might they be different?

Probability

Teacher Name: _____

Student Name: _____

CATEGORY	4	3	2	1	Score
Contributions	Routinely provides useful ideas when participating in the group and in classroom discussion. A definite leader who contributes a lot of effort.	Usually provides useful ideas when participating in the group and in classroom discussion. A strong group member who tries hard!	Sometimes provides useful ideas when participating in the group and in classroom discussion. A satisfactory group member who does what is required.	Rarely provides useful ideas when participating in the group and in classroom discussion. May refuse to participate.	
Attitude	Never is publicly critical of the project or the work of others. Always has a positive attitude about the task(s).	Rarely is publicly critical of the project or the work of others. Often has a positive attitude about the task(s).	Occasionally is publicly critical of the project or the work of other members of the group. Usually has a positive attitude about the task(s).	Often is publicly critical of the project or the work of other members of the group. Often has a negative attitude about the task(s).	
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.	
Strategy/ Procedures	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems	
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s).	