Math 445/545 Homework - Branching Processes
Also: Chapter 4 Problems 64, 65, 66

1. Suppose a parent has no offspring with probability 1/4 and two offspring with probability 3/4. Find expected size of the k^{th} generation, $k = 1, 2, 3, 4, 5$. What is the probability that the family line eventually dies out?

2. A population begins with a single individual. In each generation, each individual dies with probability 1/2 or doubles with probability 1/2. Let X_n denote the number of individuals in the population in the n^{th} generation. Find the mean and variance of X_n.

3. A population begins with a single individual. In each generation, each individual dies with probability 2/3 or doubles with probability 1/3. Let X_n denote the number of individuals in the population in the n^{th} generation. Find the probability of extinction.

4. At each stage of an electron multiplier, each electron, upon striking the plate, generates a Poisson distributed number of electrons for the next stage. Suppose the mean of the Poisson distribution is λ. Determine the mean and variance for the number of electrons in the n^{th} stage.

5. Families in a certain society choose the number of children they will have according to the following rule: If the first child is a girl, they have exactly one more child. If the first child is a boy, they continue to have children until the first girl, and then cease childbearing.

   (a) For $k = 0, 1, 2, \ldots$ what is the probability that a particular family will have k children in total?

   (b) For $k = 0, 1, 2, \ldots$ what is the probability that a particular family will have exactly k male children among their offspring?

1. $\mu = 3/2, \mu^2, \mu^3, \ldots, \pi_0 = 1/3$.

2. $E[X_n] = 1, Var(X_n) = n, \forall n$.

3. $\mu = \frac{1}{3} < 1 \Rightarrow \pi_0 = 1$.

4. $E[X_n] = \lambda^n$; if $\lambda = 1$ then $Var(X_n) = n\lambda = n$, otherwise $Var(X_n) = \lambda^n(1-\lambda^n)/(1-\lambda)$.

5. (a) $p_0 = p_1 = 0; p_2 = 3/4; p_k = (1/2)^k, k \geq 3$. (b) P(0 males)=1/4, P(1 male)=1/2, P(k males)= $(1/2)^{k+1}, k \geq 2$.

64 $1/(1-\mu)$.

66 (a) $\pi_o = 1/3$ (b) $\pi_0 = 1$, (c) $\pi_0 = (\sqrt{3}-1)/2$.