Types of Insurance:

1. Whole life: pays when the insured dies.
2. n year term insurance: pays if the insured dies within n years. If the insured survives n years, no benefit is paid.
3. n year pure endowment: pays if the insured survives n years. If the insured dies within n years, no benefit is paid.
4. n year endowment insurance: Combines term insurance and pure endowment. Pays if the insured dies within n years, or pays if insured suvives n years.

Life Insurance Notation The "big A" symbol denotes the actuarial present value (APV) of \$1 of insurance, issued to a life age x, assuming constant force of interest.

Table 1: Notation for APV of \$1 of standard insurance.

Symbol	Type of Insurance
$\bar{A}_x$	Whole life insurance payable at moment of death
A_x	Whole life insurance payable at end of year of death
${}_{m\vert}\bar{A}_x$	Whole life insurance payable at moment of death, deferred m years
$\bar{A}^1_{x:\overline{n}\vert}$	n year term life insurance payable at moment of death
$A^1_{x:\overline{n}\vert}$	n year term life insurance payable at end of year of death
$A_{x:\overset{1}{\overline{n}\vert}}$	n year pure endowment
${}_nE_x$	n year pure endowment
$\bar{A}_{x:\overline{n}\vert}$	n year endowment insurance payable at moment of death, or upon survival n years
$A_{x:\overline{n}\vert}$	n year endowment insurance payable at end of year of death, or upon survival n years

Basic formula: insurances payable at the end of year of death

$$APV = E[Z] = \sum [\text{ present value of payment at time k+1} \times \text{prob. payment is made at time k+1}]$$

Basic formula: insurances payable at the moment of death

$$APV = E[Z] = \int [\text{ present value of payment at time t} \times \text{ density at time t}]$$

Recall that the density of $T(x)$ is ${}_tp_x\, \mu(x+t)$.

Variance The "big A" symbol with a left superscript of 2 indicates that the calculation of APV is at twice the force of interest. Under constant force of interest, $E[Z^2]$ is found by computing E[Z] for the same type of insurance at double the force of interest, where Z is the present value random variable. For example, under constant force of interest,

$$Var(Z) = E[Z^2] - (E[Z])^2 = {}^2\bar{A}_x - \bar{A}_x^2$$

gives the variance of the APV of a unit of whole life insurance issued to (x) payable at the moment of death.

Accumulated and present values, interest only: Accumulation function, constant force of interest (compound interest with constant rate of effective annual interest): $a(t)$ is the accumulated value of \$1 at time t, invested at constant force of interest δ or constant effective annual rate of interest i or constant nominal annual rate of interest $i^{(m)}$.

$$a(t) = e^{\delta t} \qquad \text{(continuous compounding)}$$
$$a(n) = (1+i)^n \qquad \text{(annual compounding)}$$
$$a(n) = (1+\frac{i^{(m)}}{m})^{mn} \qquad \text{(compounding m periods annually)}$$

The present value of \$1 assuming constant compound interest:

$$a(t) = e^{-\delta t} \qquad \text{(continuous compounding)}$$
$$a(n) = (1+i)^{-n} \qquad \text{(annual compounding)}$$
$$a(n) = (1+\frac{i^{(m)}}{m})^{-mn} \qquad \text{(compounding m periods annually)}$$

Selected computing formulas:

$$\bar{A}_x = \int_0^\infty v^t \, {}_tp_x \, \mu(x+t)dt$$

$$\bar{A}^1_{x:\overline{n}|} = \int_0^n v^t \, {}_tp_x \, \mu(x+t)dt$$

$$A_x = \sum_{k=0}^{\infty} v^{k+1} \, {}_{k|}q_x = \sum_{k=0}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k}$$

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} \, {}_{k|}q_x = \sum_{k=0}^{n-1} v^{k+1} \, {}_kp_x \, q_{x+k}$$

$$A_{\underset{x:\overline{n}|}{\;\;1}} = {}_nE_x = v^n \, {}_npx$$

Relationship between term insurance and endowment insurance:

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ \ 1}$$

$$\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ \ 1}$$

Note that the pure endowment portion of the endowment insurance is the same under the continuous or discrete model.

Actuarial discount factor The APV of an n-year pure endowment ${}_nE_x$ gives the APV of \$1 payable in n years. That is,

$${}_nE_x = A_{x:\overline{n}|}^{\ \ \ 1} = v^n {}_np_x$$

So ${}_nE_x$ is the *discount factor* when both interest and mortality are considered in the present value of a contingent future payment. To bring a future payment of \$1 to (actuarial) present value, multiply by ${}_nE_x$. To accumulate \$1 under interest and mortality, divide by ${}_nE_x$.

Deferred insurances For an insurance with an m year deferral period, no benefit is paid in the first m years. The payment is valued at the present time but the insurance coverage begins m years later. To compute the APV of this type of insurance, use the discount factor ${}_mE_x$ to bring the payment to the present time. For example,

$${}_{m|}A_x = {}_mE_x\, A_{x+m}.$$

That is, an m year deferred whole life insurance issued to (x) has APV at time m equal to that of a whole life insurance issued to a life aged x+m. To bring the value to time 0, multiply by ${}_mE_x$.

Recursion Formulas

1. A whole life insurance is equivalent to a one year term insurance plus a one-year deferred whole life insurance.

$$A_x = vq_x + vp_x\, A_{x+1} = A^1_{x:\overline{1}|} + E_x A_{x+1}$$

2. A whole life insurance is equivalent to an n year term insurance plus an n year deferred whole life insurance.

$$A_x = A^1_{x:\overline{n}|} + {}_{n|}A_x = A^1_{x:\overline{n}|} + v^n {}_np_x\, A_{x+n} = A^1_{x:\overline{n}|} + {}_nE_x A_{x+n}$$

3. An n year term insurance is equivalent to an m year term insurance plus an n-m year term insurance deferred m years.

$$A^1_{x:\overline{n}|} = A^1_{x:\overline{m}|} + {}_{m|}A^1_{x+m:\overline{n-m}|} = A^1_{x:\overline{m}|} + {}_mE_x A_{x+m:\overline{n-m}|}$$