Table 1: Notation for present value of annuities certain.

Symbol	Type of Annuity
$\bar{a}_{\overline{n}\|}$	n year annuity certain paid continuously
$\bar{a}_{\overline{\infty}\|}$	perpetuity paid continuously
$a_{\overline{n}\|}$	n year annuity certain with payments at end of years (annuity immediate)
$\ddot{a}_{\overline{n}\|}$	n year annuity certain with payments at beginning of years (annuity due)

Table 2: Notation for actuarial present value of life annuities.

Symbol	Type of Annuity
$\bar{a}_x$	Whole life annuity paid continuously
a_x	Whole life annuity immediate with annual payments at end of years
$\ddot{a}_x$	Whole life annuity due with annual payments at beginning of years
${}_{m\|}\bar{a}_x$	Whole life annuity paid continuously, deferred m years
$\bar{a}_{x:\overline{n}\|}$	n year temporary life annuity paid continuously
$a_{x:\overline{n}\|}$	n year temporary life annuity with payments at end of years
$\ddot{a}_{x:\overline{n}\|}$	n year temporary life annuity with payments at beginning of years

Interest only (annuities certain)

$$a_{\overline{n}|} = \frac{1-v^n}{i} \qquad v = \frac{1}{1+i}$$

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d} \qquad d = 1-v$$

$$s_{\overline{n}|} = \frac{(1+i)^n-1}{i} \qquad \frac{v}{1-v} = \frac{1}{i}$$

$$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n-1}{d} \qquad d = \frac{i}{1+i}$$

$$\bar{a}_{\overline{t}|} = \frac{1-v^t}{\delta} \qquad v = e^{-\delta}$$

Formulas for m^{thly} compounding: compound interest with constant rate of effective annual interest i or constant rate of annual discount d, nominal annual rate of interest $i^{(m)}$ or nominal annual rate of discount $d^{(m)}$.

$$1+i = (1+\frac{i^{(m)}}{m})^m \qquad \text{(compounding m periods annually)}$$

$$1-d = (1-\frac{d^{(m)}}{m})^m \qquad \text{(compounding m periods annually)}$$

Life annuities (interest plus mortality)

Whole life annnuity due: if the curtate future lifetime of (x) is K, then K+1 payments are made beginning at time 0, and the present value random variable is $1 + v + v^2 + \cdots + v^K = \ddot{a}_{\overline{K+1|}}$.

$$\ddot{a}_x = E[\ddot{a}_{\overline{K+1|}}] = E\left[\frac{1-v^{K+1}}{d}\right] = \frac{1-E[v^{K+1}]}{d} = \frac{1-A_x}{d}.$$

The variance of the present value random variable is

$$Var(\ddot{a}_{\overline{K+1|}}) = Var\left(\frac{1-v^{K+1}}{d}\right) = \frac{Var(v^{K+1})}{d^2} = \frac{{}^2A_x - A_x^2}{d^2}.$$

If complete future lifetime of (x) is T, then 1 is paid continuously until time T and the present value random variable is $\bar{a}_{\overline{T|}}$ (computed using the formula for annuity certain).

$$\bar{a}_x = E[\bar{a}_{\overline{T|}}] = E\left[\frac{1-v^T}{\delta}\right] = \frac{1-E[v^T]}{\delta} = \frac{1-\bar{A}_x}{\delta}.$$

$$Var(\bar{a}_{\overline{T|}}) = Var\left(\frac{1-v^T}{\delta}\right) = \frac{Var(v^T)}{\delta^2} = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2}$$

Relation between life annuities and insurance:

$$\ddot{a}_x = \frac{1-A_x}{d} \qquad \ddot{a}_{x:\overline{n|}} = \frac{1-A_{x:\overline{n|}}}{d}$$

$$a_x = \ddot{a}_x - 1 = \frac{1-(1+i)A_x}{i}$$

$$\bar{a}_x = \frac{1-\bar{A}_x}{\delta} \qquad \bar{a}_{x:\overline{n|}} = \frac{1-\bar{A}_{x:\overline{n|}}}{\delta}$$

Current payment technique:

Life annuities due

$$APV = E[Y] = \sum_k v^k {}_kp_x = \sum[\text{PV of payment at time k} \times \text{prob. payment is made at time k}]$$

Life annuities immediate

$$APV = E[Y] = \sum_k v^{k+1} {}_{k+1}p_x = \sum[\text{PV of payment at time k+1}\times\text{prob. payment is made at time k+1}]$$

Life annuities payable continuously

$$APV = E[Y] = \int v^t {}_tp_x dt = \int[\text{PV of payment at time t} \times \text{ prob. payment is made at time t}]dt$$