### Table 1: Notation for present value of annuities certain.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type of Annuity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{a}_n )</td>
<td>n year annuity certain paid continuously</td>
</tr>
<tr>
<td>( \overline{a}_{\infty} )</td>
<td>perpetuity paid continuously</td>
</tr>
<tr>
<td>( a^n )</td>
<td>n year annuity certain with payments at end of years (annuity immediate)</td>
</tr>
<tr>
<td>( \ddot{a}_n )</td>
<td>n year annuity certain with payments at beginning of years (annuity due)</td>
</tr>
</tbody>
</table>

### Table 2: Notation for actuarial present value of life annuities.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type of Annuity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{a}_x )</td>
<td>Whole life annuity paid continuously</td>
</tr>
<tr>
<td>( a_x )</td>
<td>Whole life annuity immediate with annual payments at end of years</td>
</tr>
<tr>
<td>( \ddot{a}_x )</td>
<td>Whole life annuity due with annual payments at beginning of years</td>
</tr>
<tr>
<td>( m\overline{a}_x )</td>
<td>Whole life annuity paid continuously, deferred m years</td>
</tr>
<tr>
<td>( \overline{a}_{x:n} )</td>
<td>n year temporary life annuity paid continuously</td>
</tr>
<tr>
<td>( a_{x:n} )</td>
<td>n year temporary life annuity with payments at end of years</td>
</tr>
<tr>
<td>( \ddot{a}_{x:n} )</td>
<td>n year temporary life annuity with payments at beginning of years</td>
</tr>
</tbody>
</table>

### Interest only (annuities certain)

\[
\begin{align*}
\overline{a}_n &= \frac{1 - v^n}{i} \\
\ddot{a}_n &= \frac{1 - v^n}{d} \\
\overline{s}_n &= \frac{(1+i)^n - 1}{i} \\
\ddot{s}_n &= \frac{(1+i)^n - 1}{d} \\
\overline{a}_t &= \frac{1 - v^t}{\delta} \\
\end{align*}
\]

Formulas for \( m^{thly} \) compounding: compound interest with constant rate of effective annual interest \( i \) or constant rate of annual discount \( d \), nominal annual rate of interest \( i^{(m)} \) or nominal annual rate of discount \( d^{(m)} \).

\[
\begin{align*}
1 + i &= (1 + \frac{i^{(m)}}{m})^m & \text{(compounding m periods annually)} \\
1 - d &= (1 - \frac{d^{(m)}}{m})^m & \text{(compounding m periods annually)}
\end{align*}
\]
Life annuities (interest plus mortality)

Whole life annuity due: if the curtate future lifetime of (x) is K, then K+1 payments are made beginning at time 0, and the present value random variable is \( 1 + v + v^2 + \cdots + v^K = \ddot{a}_{K+1} \).

\[
\ddot{a}_x = E[\ddot{a}_{K+1}] = E \left[ \frac{1 - v^{K+1}}{d} \right] = \frac{1 - E[v^{K+1}]}{d} = \frac{1 - A_x}{d}.
\]

The variance of the present value random variable is

\[
Var(\ddot{a}_{K+1}) = Var \left( \frac{1 - v^{K+1}}{d} \right) = Var(v^{K+1}) = \frac{2A_x - A_x^2}{d^2}.
\]

If complete future lifetime of (x) is T, then 1 is paid continuously until time T and the present value random variable is \( \ddot{a}_T \) (computed using the formula for annuity certain).

\[
\ddot{a}_x = E[\ddot{a}_T] = E \left[ \frac{1 - v^T}{\delta} \right] = \frac{1 - E[v^T]}{\delta} = \frac{1 - \ddot{A}_x}{\delta}.
\]

\[
Var(\ddot{a}_T) = Var \left( \frac{1 - v^T}{\delta} \right) = Var(v^T) = \frac{2\ddot{A}_x - \ddot{A}_x^2}{\delta^2}.
\]

Relation between life annuities and insurance:

\[
\ddot{a}_x = \frac{1 - A_x}{d} \quad \ddot{a}_{x:|n]} = \frac{1 - A_{x:|n]}}{d}
\]

\[
a_x = \ddot{a}_x - 1 = \frac{1 - (1 + i)A_x}{i} \quad a_{x:|n]} = \frac{1 - \ddot{A}_{x:|n]}{\delta}
\]

\[
\ddot{a}_x = \frac{1 - \ddot{A}_x}{\delta} \quad \ddot{a}_{x:|n]} = \frac{1 - \ddot{A}_{x:|n]}{\delta}
\]

Current payment technique:

Life annuities due

\[
APV = E[Y] = \sum_k v^k k p_x = \sum [PV of payment at time k \times prob. payment is made at time k]
\]

Life annuities immediate

\[
APV = E[Y] = \sum_k v^{k+1} k+1 p_x = \sum [PV of payment at time k+1 \times prob. payment is made at time k+1]
\]

Life annuities payable continuously

\[
APV = E[Y] = \int v^t p_x dt = \int [PV of payment at time t \times prob. payment is made at time t] dt
\]