Sumgo Here and Sumgo There

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THE IDEA OF "SUMGO" WAS SUGGESTED BY the game of bingo and the need to illustrate the utility of educational games, help students practice skills, and introduce new concepts. This game was designed to investigate an interesting distribution while practicing a computational skill. As a result, the activity described in this article focuses on the concepts of sample spaces and exact probabilities while providing practice in addition. In designing "sumgo," I envisioned a mathematics class actively engaged with the game while practicing addition and learning about data interpretation, experimental and theoretical probability, and the consequences of randomness.

"Sumgo"

"SUMGO" IS SIMILAR TO BINGO BUT WITH A twist. It is a fairly simple game requiring only four dice, "sumgo" cards (see **fig. 1**), and plastic chips. Although comparable to bingo in many ways, "sumgo" differs from the traditional game in the nature of the cards and the rules for covering spaces on the cards. The rules are as follows:

1. After reading the rules, each student selects numbers to be written on the squares of his or her "sumgo" card. Each of the numbers placed on the "sumgo" card must be unique, meaning that no number can be repeated in any two squares of a student's card. Additionally, the center "sumgo" square on each card should be covered with a chip.
2. The teacher rolls a pair of dice, one red and the other green, to produce the first addend of a sum in which the red die corresponds to the tens place and the green die, to the ones place. For instance, if a 1 is rolled on the red die and a 6 is rolled on the green die, the addend would be 16. The roll is repeated with a second pair of dice to produce the second addend for the sum. The first addend is called out, followed by the second addend.
3. The students compute the sum of the two addends, stating the result aloud. The teacher records this value for future verification of a winning claim.
4. If the result from step 3 appears on a student's card, then he or she places a chip on the square containing that value.
5. Play continues until a student claims that "SUMGO!" has been achieved; that is, five contiguous squares have been covered in a horizontal, vertical, or diagonal row.
6. The student who claims to have "sumgo" reads off the winning values while another student double-checks the work and the teacher verifies that all the numbers have been produced as part of the game. If an error is detected, the students continue to generate sums until another winning claim is made.

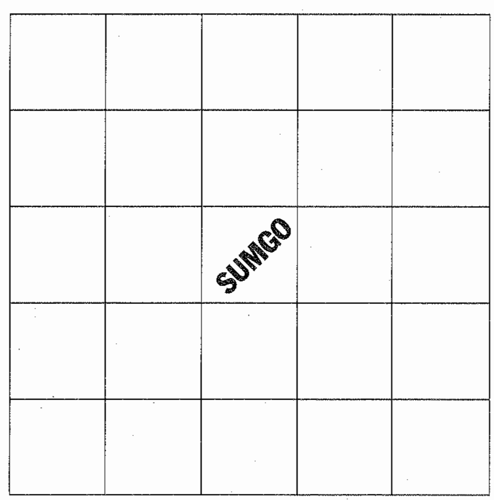

Fig. 1 "Sumgo" playing card

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As students read through the rules and understand how the values are produced, they are led to an initial investigation that sets the stage for examining aspects of probability.

The Initial "Sumgo" Investigation: What Values Are Produced?

IN SELECTING THE NUMBERS TO BE WRITTEN ON the cards, students must consider the constraints on the obtainable sums. After some investigation, students should conclude that the maximum and minimum values are 132 and 22, respectively. In addition to identifying the maximum and minimum values, students must also address the following question: "If 22 is the minimum and 132 is the maximum, then can every whole number between these values be obtained?" Although this question can be approached in various ways, many students may simply choose a few "oddball" numbers to see whether they can generate those values. This meager set of examples, although somewhat compelling to students, cannot adequately answer the question. At this point, the teacher has the opportunity to reinforce correct thinking, curb incorrect paths, and illustrate that random examples are insufficient proof. Prompting students to recognize that the investigation requires systematic examination of sample spaces is particularly important in this lesson. One method of inquiry would engage students in examining each number between 22 and 132 to determine whether it can be decomposed into a sum with addends from the set {11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66}. This preliminary investigation reveals that every number between 22 and 132 is obtainable as part of the game, and students can select any of these values to fill in their "sumgo" cards.

Students may choose "oddball" numbers to work with

After the students have recorded their chosen values on their "sumgo" cards and the chips have been distributed, the class should play several games of "sumgo." During the play, the teacher should document all the sums obtained to verify the values when a student calls out "SUMGO!" After a few games, students will notice that certain numeric values seem to come up more often than others and may begin to comment, "Hey, we already had that number!" This recognition leads to the investigation of the game's probability and the consideration of a "best" set of numbers to be placed on the "sumgo" card.

The Second "Sumgo" Investigation: What Values Appear More Often?

THE SECOND INVESTIGATION GIVES THE TEACHER an opportunity to revisit the commutative property of addition. After reviewing the fact that the sum of 34 + 56 is the same as the sum of 56 + 34, for example, the teacher can ask students to consider the ramifications of this property on their "best" sets of numbers. At this point, students begin to recognize the need for a complete examination of the distribution, not just the set of possible values.

To determine all the possible sums that generate each number between 22 and 132, divide the students into groups and ask each group to calculate the sums for portions of the range. When conducting this activity in my class of prospective elementary school teachers, I divided my thirty students into six groups (A–F). I then asked the groups to use the chart in **figure 2** to compute the possible sums for each of the numbers in their assigned sections of column 2 with all the numbers in column 3. In doing this task, each group computed a total of 6 × 36, or 216, sums.

Once students compute the sums and compile all the data, they can order the sum values to quickly organize the collected information. The sums gener-

Group	Assigned Range of Numbers		Addends	
A	11	14	11	14
	12	15	12	15
	13	16	13	16
B	21	24	21	24
	22	25	22	25
	23	26	23	26
C	31	34	31	34
	32	35	32	35
	33	36	33	36
D	41	44	41	44
	42	45	42	45
	43	46	43	46
E	51	54	51	54
	52	55	52	55
	53	56	53	56
F	61	64	61	64
	62	65	62	65
	63	66	63	66

Fig. 2 Numbers to add in "sumgo"

ated will range from 22 through 132; a histogram, similar to the one shown in **figure 3,** can be used to characterize the data. From the information in **figure 2,** students can begin to investigate the exact probabilities of obtaining specific values.

The Third "Sumgo" Investigation: What Probability Is Associated with Each Value?

STUDENTS MUST DECIDE THE BEST WAY OF using the chart in **figure 2** to determine the probability of obtaining the sum x, where $22 \le x \le 132$. Discussions should center on the number of occurrences, the number of possible answers, and the probability of obtaining a particular value. Students in my class observed that "the probability of getting a number that ends in 7 is greater than getting any other number" and "the probability of getting some numbers was much higher than that of getting others, especially in the 70s range." In addition, students should be able to make comparisons between scenarios. For example, a student in my Advanced Mathematics for Elementary Teachers class wrote that "if you choose a number with a last digit of 3 you don't have as good as chance as if the last digit was 6. To get a 3 as a last digit, the numbers must be ___1 + ___2 or vice versa. To get 6 as a last digit you can have ___1 + ___5, ___2 + ___4, or even ___3 + ___3. The probability of this is much higher." Such realizations offer the opportunity to help students translate their intuitive observations into comparisons using probability.

Renumbering the "Sumgo" Cards and Playing the Game Again

THESE TYPES OF REALIZATIONS SHOULD LEAD students into making their "best" choices for the numbers to write on their "sumgo" cards. After a few games of using these new cards, however, students tend to realize that sometimes the best is not always good enough! For example, students in my class made such comments as these:

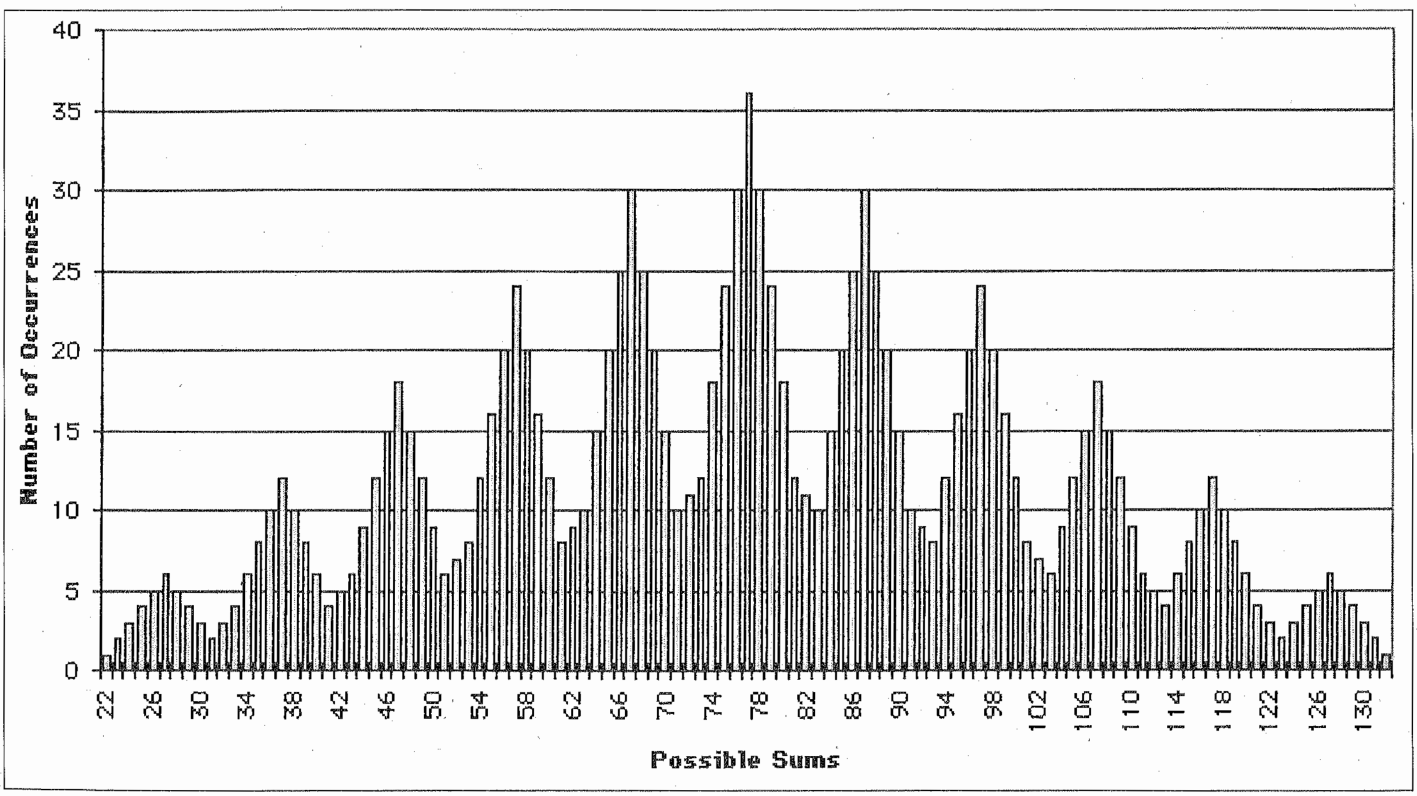
> It is based on luck and how the numbers are played. Even after finding out which numbers occur more it didn't help me to win even if I picked those numbers. It was a really fun game that I will take on with me and use in my classroom.

> Even if you write the numbers down that have the highest probability of occurring, it is still a game of chance. You never know when they will be called, so you can't arrange your card to get Sumgo first. All the numbers are arranged in different areas on the board. . . . It all "sums" to a game of lottery/chance.

Student Reactions to the Game and the Exploration into Probability

MY STUDENTS WROTE ABOUT THE "SUMGO" GAME and discussed what they had learned from the activity. In addition to identifying the arithmetic-skill context and the examination of probability inherent in the game, my students pointed to a variety of other important issues underlying the investigation:

Fig. 3 **Distribution of all possible "sumgo" sums**



The game Sumgo was so fun. . . a new game to really get everyone of all ages excited. . . . I also learned how you can make a simple concept into a game. You can use this in so many different ways and build on this idea. You can make it simpler or more complex depending on the age of your students as well as their capabilities.

I would say that the main thing I learned from Sumgo is that sometimes the best learning tools for your classroom can be ones that you think of yourself. I would say that I learned that fun activities like Sumgo are great because students are motivated. They want to play and win the game, but in the meantime they are learning and practicing their addition skills.

From the game Sumgo I learned that there are games out there that can be thought up by the teacher. . . . Tying a fun activity into any lesson will give the children a chance to relax, enjoy themselves, and learn (practice) something all at the same time.

"The best tools are the ones you think of yourself"

Sumgo showed me that I can get my students to review their facts without mindless repetitive worksheets . . . a great motivator for students to learn their math facts . . . a great educational game that students will love.

I actually played Sumgo with my 2nd graders that I tutored. (I hope you don't mind.) I liked how you had to add up the #'s, and find the # like bingo. I learned to teach the students that you can learn while having fun. Great game!

These prospective elementary school teachers obviously recognized the important components of the game and saw the utility of making mathematics learning fun. Such lessons might have been hard to convey in a meaningful way without sharing "sumgo" as a classroom activity with these prospective teachers.

Conclusion

"SUMGO" IS ONE EXAMPLE OF A SIMPLE GAME that is useful for illustrating connections in mathematics, particularly concepts related to probability, while practicing elementary mathematical skills. The most powerful aspects of this game lie in its simplicity and its ability to encourage students to investigate mathematical concepts in an exciting and engaging manner. Instead of serving as a "Friday filler," the activity described in this article proves that a game can meet specific student objectives while promoting an atmosphere of fun. Additionally, such games allow teachers to make minor adjustments and investigate new possibilities. For instance, a simple variation of "sumgo" might be "minusgo," using the underlying operation of subtraction. This subtle adjustment would force students to consider both signed numbers and the fact that the commutative property does not apply in subtraction, that is, $46 - 23 \neq 23 - 46$. These elements do not affect the distribution's shape but provide occasions for added exploration.

This game also introduces other questions and investigations. For example, after the students have filled in their cards and the chips are about to be distributed, the teacher can ask, "What is the minimum number of chips you need so that if you used that many, you would know for certain that you had to have 'sumgo'?" This question can lead students into investigating covering spaces. Another question, although possibly not one that middle school students could address, is "Does a 'best' arrangement of the numbers with highest probability exist, and what might that arrangement be?" Advanced students could use a spreadsheet program to develop simulations to test the prospective cards. This game and related permutations allow the teacher to investigate a variety of issues that are directly connected with probabilistic thinking, as well as peripheral concepts that are appropriate for both middle school students and prospective teachers.

Games offer the possibility of engaging students in activities that contain rich explorations into complex mathematical concepts. Not every game promotes such exploration, however, so teachers must be as selective in choosing games as they are in selecting any other instructional material or method. Games such as "sumgo" address the content and instructional concerns raised by the National Council of Teachers of Mathematics in both its *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and its *Professional Standards for Teaching Mathematics* (NCTM 1991). According to my students, the "sumgo" game is engaging for students from elementary school to college. Instead of presenting routine tasks, games accomplish the goals of encouraging mathematically literate and lifelong learners while placing students in environments that are conducive to developing communal problem-solving skills.

References

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: NCTM, 1989.

———. *Professional Standards for Teaching Mathematics*. Reston, Va.: NCTM, 1991.