For bivariate normal distribution with mean $\vec{\mu}$ and covariance matrix C, the PDF is given by:

$$\phi_2(\vec{x}) = \frac{1}{2\pi|C|^{1/2}} \exp\Big(\frac{-1}{2}(\vec{x}-\vec{\mu})^T C^{-1}(\vec{x}-\vec{\mu})\Big)$$

In the special case of zero mean $\vec{\mu} = 0$ and C is a "correlation matrix" Ω, i.e.,

$$\Omega = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix},$$

this simplifies (writing $\vec{x}$ as (x, y)) to

$$\phi_2(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\Big(\frac{-(x^2 - 2\rho xy + y^2)}{2(1-\rho^2)}\Big)$$

Then the skew-normal PDF f is given by

$$f(x, y) = 2\phi_2(x, y)\mathbf{\Phi}(\alpha_1 x + \alpha_2 y)$$

where $\mathbf{\Phi}$ is the $N(0, 1)$ CDF.